

Signed Frequency Certificates and Constraint Costs for Odd Weil Forms

Scott Schubr
Luminary Oddities

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Abstract

We construct complete signed-frequency certificates for odd Weil forms and prove $q_H[h] \geq -5.289 \cdot 10^{-19} \|h\|_2^2$ on the original polar-constrained minimal domain in the reciprocal-theta norm. At arithmetic cutoff sixteen, an exact eleven-null reduction and 206 disjoint positive-frequency allocations pay all 286 negative observations. The 614-direction head retains every mixed covariance with the positive sources and all twelve physical constraint rows. Exact dyadic receiving maps, a full congruence and directed matrix signs certify the complete infinite-dimensional return. A common Schur row bounds the omitted arithmetic shifts; continuous approximation, complete theta kernels, quadrature, root strips and infinite tails are paid in their stated metrics. We also prove degree-independent Fourier locality, quantitative growing-cutoff constraint costs and exact-coordinate precision bounds. The accompanying reader reconstructs the complete source Grams and signed tests from compact source moments.

1 Complete certification and its resource costs

A lower bound for a Weil form depends on the recipient norm, the domain and the complete signed operator. We work in the reciprocal-theta realization of RH , *Approximately* [1]. Its earlier allowance $1.5 \cdot 10^{-11}$ supplies a published benchmark in the same original norm. The present construction combines actual negative-frequency observations with disjoint positive-frequency references and an exact finite null reduction.

Theorem 1.1 (Complete original-domain certificate). *For the original polar-constrained minimal odd Weil form on V ,*

$$q_H[h] \geq -5.289 \cdot 10^{-19} \|h\|_2^2, \quad h \in V.$$

At arithmetic cutoff $M = 16$, 206 positive-frequency allocations pay all 286 negative observations. The complete receiving, approximation and raw arithmetic payments have sum less than $5.288050501026843 \cdot 10^{-19}$ in this same norm.

The first 46 observations use the exact orthogonal complement of eleven genuine complete-form null vectors and the original polar row. Their 614-direction receiving test retains the complete negative-positive covariance and every one of the 2294×12 source-constraint entries. The other 240 observations have 205 ambient odd-Hilbert receiving inequalities. Applying all comparisons to the same genuine-null projection and using disjoint frequency ownership gives the original-domain theorem. An exact source-aware congruence and a fixed dyadic receiving map permit a smaller regularization through an affine full-matrix test.

The construction also quantifies resources along growing cutoffs. For $\kappa_j = 2^{-j}$ and $M_j = \lceil j/2 \rceil + 10$, the complete positive reference on any additional subspace of codimension $D_j \leq j/(64 \log j)$ satisfies

$$\Lambda_j \geq 1 - 2e^{-(\log 2 - 1/8)j}, \quad j \geq 2^{24},$$

when the negative observation error is at most κ_j^2 . Theorem 7.4 permits the constraint space to change with the cutoff. Complete physical and receiving Gram floors then give exact coordinates with $O(j)$ precision on this dimension scale.

Frequency projections, Schur complements, contour estimates, analytic quadrature and Gram conditioning are classical tools. Their source-specific combination here supplies a complete changed-cutoff certificate, exact original-domain transport, degree-independent locality and quantitative constraint/representation costs. The negative-band approximation [2], original lower-edge realization [3] and exact complete-form null theorem [4] are retained with their actual roles. Related complementary-reference and resolvent geometry appear in [7, 8, 5].

2 Original norm, domain and complete signed split

Use

$$\widehat{f}(\xi) = \int_{\mathbb{R}} f(u) e^{-i\xi u} du, \quad \|f\|_2^2 = \int_{\mathbb{R}} |\widehat{f}(\xi)|^2 \frac{d\xi}{2\pi}.$$

On $u \geq 0$ define the complete theta density

$$\psi(u) = e^{u/2} \sum_{\ell \geq 1} (4r_\ell^2 - 6r_\ell) e^{-r_\ell}, \quad r_\ell = \pi \ell^2 e^{2u}, \quad s(u)^2 = \frac{\psi(u)}{\cosh(u/2)}. \quad (1)$$

Theta inversion makes ψ even and positive. Put $Y = L_{\text{odd}}^2(\mathbb{R})$, and let $X \subset Y$ be the closed subspace defined by the polar row

$$\langle b, h \rangle = 0, \quad b(u) = s(u) \sinh(u/2). \quad (2)$$

The complete theta envelope gives $b \in Y$. Writing $f = sh$ yields $\|h\|_2^2 = \int_{\mathbb{R}} |f|^2 \cosh(u/2) / \psi(u) du$. All statements extend to the complexification. Associated constrained forms and operators are used on their stated spaces.

Write $\Psi_0 = \Gamma'/\Gamma$ and

$$a(\xi) = \Re \Psi_0(1/4 + i\xi/2) - \Psi_0(1/4) \geq 0, \quad m_0 = \Psi_0(1/4) - \log \pi < 0, \quad c_n = \frac{\Lambda(n)}{\sqrt{n}}.$$

The digamma series proves $a \geq 0$. Start on smooth compactly supported odd functions with exact row (2). The original domain V is their completion for

$$\|h\|_V^2 = \|h\|_2^2 + \int_{\mathbb{R}} a(\xi) |\widehat{sh}(\xi)|^2 \frac{d\xi}{2\pi}.$$

This is the inherited minimal realization [3, 1]. Its complete form on the core is

$$\begin{aligned} q_H[h] &= \int_{\mathbb{R}} (m_0 + a(\xi)) |\widehat{sh}(\xi)|^2 \frac{d\xi}{2\pi} \\ &\quad - \sum_{n \geq 2} c_n \langle sh, (S_{\log n} + S_{-\log n}) sh \rangle, \quad S_b f(u) = f(u + b). \end{aligned} \quad (3)$$

The polar contribution vanishes on (2). Section 5 proves norm convergence of the raw weighted shift sum. For integer $M \geq 2$ put

$$m_M(\xi) = m_0 + a(\xi) - 2 \sum_{2 \leq n \leq M} c_n \cos(\xi \log n), \quad (4)$$

$$p_M = (m_M)_+, \quad n_M = (-m_M)_+.$$

All prime powers are retained. Let

$$P_M[h] = \int_0^\infty p_M |\widehat{sh}|^2 \frac{d\xi}{\pi}, \quad N_M[h] = \int_0^\infty n_M |\widehat{sh}|^2 \frac{d\xi}{\pi}.$$

Oddness accounts for $d\xi/\pi$. The complete signed split is

$$q_H = P_M - N_M + W_{>M}. \quad (5)$$

Both $p_M - a$ and $m_M - a$ are bounded. Thus P_M has domain V , and N_M is bounded. The split has the original completion [2].

2.1 Genuine nulls and exact original-domain transport

The exact-null theorem [4, Section 3, Lemma 3.1] gives

$$z_{2i+1} = \frac{(D^2 - 1/4)D^{2i+1}\psi}{s}, \quad D = d/du, \quad i = 0, 1, \dots \quad (6)$$

In that source $P_0 = D^2 - 1/4$ and $\phi_k = P_0 D^k \psi / s$; the present row is directly $z_{2i+1} = \phi_{2i+1}$. Each belongs to $D(H) \cap V$, has exact polar moment zero, and satisfies the operator identity $H z_{2i+1} = 0$. The cited proof uses $P_0(\chi_R D^k \psi)$ with compact smooth cutoffs, complete-theta tails and $a(\xi) \leq 17\xi^2$ to return to the original minimal form domain. The explicit formula gives zero pairing with every constrained core test; continuity and form representation give the operator-null statement. Appendix B pays the explicit graph-norm estimates used below.

Lemma 2.1 (Finite-null transport). *Let K be any finite span of the rows (6), and let Q_K be the orthogonal projection off K in the original Hilbert metric. For every $h \in V$,*

$$Q_K h \in V, \quad \|Q_K h\|_2 \leq \|h\|_2, \quad q_H[Q_K h] = q_H[h].$$

Proof. The removed vector is a finite linear combination of elements of $V \cap X$, so the projection preserves V and its polar condition. For $z \in K$, the operator-null identity gives $q_H(z, h) = \langle H z, h \rangle = 0$ for every $h \in V$. Expanding the quadratic form proves its invariance. Hilbert orthogonal projection contracts the original norm. $\square$

For the executed head take $K = K_{11} = \text{span}(z_1, z_3, \dots, z_{21})$. The polar row b implements the constraint X ; only the genuine null rows are used in Lemma 2.1. This distinction will also be explicit in the twelve-row source construction.

3 Finite receiving with positive-frequency references

For a nonnegative weight w supported in a bounded frequency set put

$$(T_w h)(\xi) = \sqrt{w(\xi)} \int_{\mathbb{R}} s(u) h(u) \sin(\xi u) du \quad \text{in } L^2(\mathbb{R}_+, d\xi/\pi).$$

For the numerical construction this map acts on the ambient odd Hilbert space $Y = L^2_{\text{odd}}(\mathbb{R})$; its inequalities can then be compressed to X . In particular $N_M = T_{n_M}^* T_{n_M}$ on Y . On frequency intervals take $z_k = \mathbf{1}_I \sqrt{w} T_k(t_I)$, where $t_I : I \rightarrow [-1, 1]$ is affine. For their column map Z , define the raw metric $D = Z^* Z$ and $E = T_w^* Z$.

Lemma 3.1 (Frequency range projection). *For positive-weight columns with metric D_+ , ridge $\rho_+ > 0$ and $W = T_{p_M}^* Z_+$,*

$$P_{\text{fin}} = W(D_+ + \rho_+ I)^{-1} W^* \leq P_M.$$

Such constructions add to at most P_M when their frequency supports have disjoint interiors.

Proof. Singular values give $0 \leq Z(Z^* Z + \rho I)^{-1} Z^* \leq I$. Conjugate by the observation map. For disjoint sets their exact observation energies add, and each finite projection is below its own energy. $\square$

Absorb both ridges into $D_n, D_p > 0$. Set

$$N_\star = E D_n^{-1} E^*, \quad P_{\text{fin}} = W D_p^{-1} W^*, \quad G_{nn} = E^* E, \quad C = E^* W, \quad G_{pp} = W^* W.$$

These are full ambient mixed covariances. An ambient operator inequality restricts exactly to X . The head uses the complete exact-row compression proved below.

Theorem 3.2 (Full mixed variational certificate). *Let $\kappa > 0$, $L = G_{pp} + \kappa D_p$, and fix a matrix F of size $(\dim D_p) \times (\dim D_n)$. The condition*

$$\kappa D_n - G_{nn} + C F + F^* C^* - F^* L F \geq 0 \tag{7}$$

implies $N_\star \leq P_{\text{fin}} + \kappa I$. If an approximate full Gram satisfies

$$-\delta \text{diag}(D_n, D_p) \leq G - \tilde{G} \leq \delta \text{diag}(D_n, D_p), \tag{8}$$

it suffices to check

$$\begin{aligned} \kappa D_n - \tilde{G}_{nn} + \tilde{C} F + F^* \tilde{C}^* - F^* (\tilde{G}_{pp} + \kappa D_p) F \\ - \delta (D_n + F^* D_p F) \geq 0. \end{aligned} \tag{9}$$

Proof. The positive inverse identity gives

$$\kappa E^* (\kappa I + W D_p^{-1} W^*)^{-1} E = G_{nn} - C L^{-1} C^*.$$

Completing the square,

$$C L^{-1} C^* - (C F + F^* C^* - F^* L F) = (L^{-1} C^* - F)^* L (L^{-1} C^* - F) \geq 0.$$

Thus (7) implies $E^* (\kappa I + P_{\text{fin}})^{-1} E \leq D_n$. Singular values of $(\kappa I + P_{\text{fin}})^{-1/2} E D_n^{-1/2}$ give the desired operator inequality. The Gram-dependent part is $-(I, -F^*) G (I, -F^*)^*$. Apply (8) to $(v, -Fv)$, giving precisely $\delta v^* (D_n + F^* D_p F) v$. This proves (9). $\square$

A dyadic F is an exact witness. A numerical inverse can propose it; the proof uses its literal entries and the enclosed sign. The approximate Gram need not be positive. All mixed covariance remains in the sign test. If an additional exact-row compression is used in another construction, its full joint correction must also be retained.

An alternative is a direct certified inverse calculation. A frozen approximate inverse of L is validated by its complete Neumann residual, which encloses L^{-1} ; a directed Schur sign then proves $\kappa D_n - G_{nn} + CL^{-1}C^* \geq 0$. This is the same inverse criterion in the proof above, without exporting an explicit variational F . The execution below uses explicit dyadic receiving maps for every group.

3.1 Complete exact-row covariance

Let $\mathcal{Z} : \mathbb{C}^r \rightarrow Y$ be an injective row map with $\Gamma = \mathcal{Z}^* \mathcal{Z} > 0$, and set

$$Q = I - \mathcal{Z}\Gamma^{-1}\mathcal{Z}^*, \quad \mathcal{C}_n = \mathcal{Z}^* E, \quad \mathcal{C}_p = \mathcal{Z}^* W, \quad \mathcal{C} = [\mathcal{C}_n, \mathcal{C}_p].$$

The complete physical source Gram on $\text{ran } Q$ is exactly

$$G_Q = G - \mathcal{C}^* \Gamma^{-1} \mathcal{C}. \quad (10)$$

Projection changes the physical columns; the true frequency metrics D_n, D_p retain their definitions.

Theorem 3.3 (Conservative row-compressed receiving). *Suppose $\bar{\Gamma} \geq \Gamma > 0$, and the ambient model obeys (8). For a fixed F , put*

$$T_F = \begin{pmatrix} I \\ -F \end{pmatrix}, \quad R_F = \mathcal{C}_n - \mathcal{C}_p F.$$

The condition

$$\kappa(D_n - F^* D_p F) - T_F^* \tilde{G} T_F + R_F^* \bar{\Gamma}^{-1} R_F - \delta(D_n + F^* D_p F) \geq 0 \quad (11)$$

implies

$$Q N_\star Q \leq Q P_{\text{fin}} Q + \kappa Q \quad \text{on } \text{ran } Q.$$

Proof. Inverse order gives $\bar{\Gamma}^{-1} \leq \Gamma^{-1}$ and hence

$$G_Q \leq G - \mathcal{C}^* \bar{\Gamma}^{-1} \mathcal{C}.$$

Thus the left side of (11) is a lower bound for $\kappa(D_n - F^* D_p F) - T_F^* G_Q T_F$. The ambient model error is paid on the whole vector $T_F v$ by (8). Apply Theorem 3.2 to QE, QW in $\text{ran } Q$. $\square$

All entries of $\mathcal{C}_n, \mathcal{C}_p$ and the proof of $\bar{\Gamma} \geq \Gamma$ belong to this certificate. Enclosing arithmetic evaluates their complete expression (11); the row source is not replaced by selected entries or by the identity Gram. Direct-kernel locality does not remove its dense correction.

Lemma 3.4 (Exact congruence sign). *Let S be a Hermitian target and let $\mathcal{R}$ be an exact invertible matrix. If an enclosing Hermitian matrix for $\mathcal{R}^* S \mathcal{R}$ has strictly positive diagonal-minus-absolute-off-diagonal row bounds, then $S > 0$.*

Proof. Gershgorin's theorem gives positivity of the entire congruence. Invertibility of $\mathcal{R}$ transfers its quadratic sign to S . An exact triangular $\mathcal{R}$ with nonzero diagonal is invertible without an accuracy assumption on the numerical proposal. $\square$

3.2 Inherited continuous negative approximation

For disjoint negative cover intervals $I_j = [a_j, b_j]$ set $\ell_j = b_j - a_j$ and $\beta_j = \int_{I_j} n_M d\xi/\pi$. With r_j Chebyshev columns, physical cutoff R and ridge ρ , the negative-band theorem [2, Sections 3–4] gives

$$0 \leq N_M - N_\star \leq \tau I, \quad \tau = \sum_j \beta_j \left[\frac{4S_0(\ell_j R/4)^{2r_j}}{(r_j!)^2} + S_{>R} \right] + \rho S_0 \sum_j \eta(r_j), \quad (12)$$

where $S_0 = \int_{\mathbb{R}} s^2 < 3$, $S_{>R} = \int_{|u|>R} s^2$, $\eta(1) = 1$ and $\eta(r) = 2$ for $r \geq 2$. For completeness, interpolation of $\sin(\xi u)$ at Chebyshev roots has error at most $2(\ell_j |u|/4)^{r_j}/r_j!$. Projection is no worse. Use the zero competitor outside $[-R, R]$. Discrete cosine Parseval bounds the squared interpolation coefficient norm by $\eta(r_j)$, paying the ridge charge. Integrate against s^2 and compress to X . The complete theta atoms give

$$S_0 \leq 16(\pi + 1)e^{-\pi}, \quad S_{>R} \leq 16(\pi e^{2R} + 1)e^{-\pi e^{2R}}. \quad (13)$$

Corollary 3.5 (Disjoint-window return). *Partition the negative observation into groups g . Let K be a finite genuine-null span as in Lemma 2.1. Suppose each approximation costs τ_g and its receiving inequality holds on $X \cap K^\perp$ with price κ_g , using mutually disjoint positive-frequency windows. The inequality may be obtained ambiently from Theorem 3.2 or on the exact recipient from Theorem 3.3. If $\|W_{>M}\| \leq \delta_M$, then*

$$q_H[h] \geq - \left(\sum_g (\kappa_g + \tau_g) + \delta_M \right) \|h\|_2^2, \quad h \in V.$$

Proof. For $h \in V$ put $u = Q_K h$. On this same u , sum the finite receiving inequalities and their complete negative approximation remainders. Lemma 3.1 bounds the sum of the allotted positive observations by P_M . Subtract the raw tail in (5). Lemma 2.1 gives $q_H[u] = q_H[h]$ and $\|u\|_2 \leq \|h\|_2$, proving the result. The empty span recovers the ambient disjoint-window return. $\square$

4 Degree-independent locality and actual source funding

Let $q = s^2$ and $\widehat{q}(\omega) = \int_{\mathbb{R}} q(u) e^{-i\omega u} du$. The ambient odd feature kernel is

$$\int_{\mathbb{R}} q(u) \sin(\xi u) \sin(\zeta u) du = \frac{1}{2} \{ \widehat{q}(\xi - \zeta) - \widehat{q}(\xi + \zeta) \}. \quad (14)$$

Double frequency integration carries $1/(2\pi^2)$. Exact row compression is applied to its full covariance by (10).

Lemma 4.1 (Complete theta Fourier locality). *For every real ω ,*

$$|\widehat{q}(\omega)| \leq \min\{32e^{-|\omega|/2}, 16384e^{-3|\omega|/4}\}.$$

If two frequency blocks start at $a_i, a_j \geq 0$, have separation d_{ij} and masses β_i, β_j , their normalized feature Gram block satisfies

$$\|G_{ij}\| \leq 16\sqrt{\beta_i \beta_j} \left(e^{-d_{ij}/2} + e^{-(a_i + a_j)/2} \right).$$

Normalization uses the true raw metrics plus nonnegative ridges; the estimates are independent of degree.

Proof. On $|\Im u| \leq 1/2$ put $r_n = \pi n^2 e^{2|\Re u|}$. Since $\cos 1 > 1/2$ and $|\cosh(u/2)| \geq (31/32) \cosh(\Re u/2)$, the complete theta series gives

$$|q(u)| \leq \frac{64}{31} \sum_{n \geq 1} (4r_n^2 + 6r_n) e^{-r_n/2}.$$

Its horizontal integral is at most

$$\frac{64}{31} \sum_{n \geq 1} (8\pi n^2 + 28) e^{-\pi n^2/2} < 32.$$

Substitute $r = \pi n^2 e^{2x}$ on both arms; integrate $(4r + 6)e^{-r/2}$ and bound the remaining scalar sum by its first terms and full geometric remainder. The complete double-exponential envelope makes the vertical contour ends vanish. Translation by $-\text{sign}(\omega)i/2$ proves the first bound.

For $|\Im u| \leq 3/4$, use $\cos(3/2) > 1/16$ and $\cos(3/8) > 9/10$. The corresponding full-series bound is

$$|q(u)| \leq \frac{20}{9} \sum_{n \geq 1} (4r_n^2 + 6r_n) e^{-r_n/16}.$$

Its integral over the two real arms is at most

$$\frac{20}{9} \sum_{n \geq 1} (64\pi n^2 + 1120) e^{-\pi n^2/16} \leq \frac{20}{9} (256S_2 + 1120S'_0),$$

where $S'_0 = \sum_{n \geq 1} e^{-3n^2/16} < 2$ and $S_2 = \sum_{n \geq 1} n^2 e^{-3n^2/16} < 15$. Indeed successive ratios give $S'_0 \leq e^{-3/16}/(1 - e^{-9/16})$, and

$$S_2 \leq \frac{32}{3e} \frac{e^{-3/32}}{1 - e^{-9/32}} < 15.$$

The integral is therefore less than 16384. The same complete contour argument at height $3/4$ proves the second bound.

A normalized source coefficient vector represents a weighted polynomial of L^2 norm at most one, since its metric contains the true raw Gram. Its weighted L^1 norm is at most $\sqrt{\beta_i}$. Apply this twice to (14); the half-strip constant becomes $32/2 = 16$. $\square$

For a union of blocks with total mass B , a uniform retained-kernel error ϵ gives normalized full-matrix error at most $B\epsilon$. The rank-one comparison $(\sqrt{\beta_i \beta_j})$ proves this for the entire coefficient vector. Complete omitted interactions are paid by the same argument and their actual minimum argument. The finite-rank correction in (10) remains dense.

4.1 The complete real kernel polynomial

The source data supply the fixed exact dyadic polynomial

$$Q(\omega) = \sum_{k=0}^{512} a_k T_k(\omega/210), \quad \sup_{|\omega| \leq 210} |\hat{q}(\omega) - Q(\omega)| < 10^{-60}. \quad (15)$$

Odd coefficients vanish exactly. Here is the complete analytic construction. The preceding strip proof gives

$$|q(u)| \leq \frac{64}{31} (256/e^2 + 24/e) \frac{e^{-3/4}}{1 - e^{-9/4}} < 128 \quad (|\Im u| \leq 1/2).$$

Use ten physical panels of width $1/4$ on $[0, 5/2]$, 128 Gauss–Legendre nodes per panel and even reflection. For real $|\omega| \leq 210$ their symmetric Gaussian error is at most

$$E_G = 2 \cdot 10 \cdot \frac{8(1/8) 128 e^{630/32} 2^{-256}}{1 - 1/2}.$$

The omitted theta atoms after eight cost $E_\theta = 5 \cdot 16\pi^2 9^4 e^{-81\pi}$, and the full real tail is bounded by

$$E_\infty = \frac{480 e^{4U - \pi e^{2U}}}{2\pi e^{2U} - 4}, \quad U = 5/2.$$

Every enclosing Fourier node receives these complete errors. On the parameter-2 frequency ellipse, $|\Im \omega| < 158$ and

$$|\widehat{q}(\omega)| \leq 480 \int_0^\infty e^{162u - 3e^{2u}} du \leq \frac{240 \cdot 80!}{3^{81}} =: M_Q.$$

Chebyshev coefficient bounds and complete Lobatto aliasing cost $4M_Q 2^{-512}$. Add the sum of all DCT coefficient radii when replacing them by exact dyadic midpoints. Odd coefficient balls contain zero before symmetry is imposed. The total is below $5.119 \cdot 10^{-63}$, establishing (15). This separates analytic integration from coefficient approximation; compare [6].

The executed head uses this Q_{512} on $[-210, 210]$. Local evaluation first expands the *fixed exact polynomial* at a nearby integer center and keeps degree 256, then shifts to the actual fractional center and keeps degree 192. The two omitted polynomial parts are paid separately. Let $M_{24} < 1.26 \cdot 10^{44}$ bound Q on every radius-24 circle centered on $[-210, 210]$. The Bernstein ellipse with semimajor $1 + 24/210$ contains these disks, giving this bound directly from the literal coefficients. The fractional center shift has modulus at most $1/2$, and the actual convolution halfwidth is at most 4. Thus the first real remainder is at most

$$E_{\text{real}} = M_{24} \frac{(4.5/24)^{257}}{1 - 4.5/24}.$$

Cauchy’s coefficient bound gives modulus at most $M_{24}/(1 - 16.5/24)$ for the intermediate degree-256 polynomial on the radius-16 disk about the actual center. The second remainder is at most

$$\frac{M_{24}}{1 - 16.5/24} \frac{(1/4)^{193}}{1 - 1/4}.$$

Their sum is below $3.411 \cdot 10^{-72} < 10^{-70}$. This is a bound for two polynomial truncations, independent of the real-kernel error in (15).

The head retains every real kernel argument through 210. An omitted term has minimum argument at least 202. Consequently the normalized source debit

$$2B \left(16384 e^{-3 \cdot 202/4} + 10^{-60} + 10^{-70} \right) \quad (16)$$

pays its complete kernel, far-image and local-model error. The true raw metrics and their enclosing entry errors are retained; the ridges dominate the model-metric discrepancies. The factor 2 safely covers the common-metric transport. It is paid on $(v, -Fv)$, without a further inverse-ridge factor.

4.2 True symbol integration and source classes

The holomorphic symbol used for source quadrature is

$$m_M(z) = \frac{\Psi_0(1/4 + iz/2) + \Psi_0(1/4 - iz/2)}{2} - \log \pi - 2 \sum_{n \leq M} c_n \cos(z \log n).$$

Complete root brackets have opposite endpoint signs and a strict derivative sign. Every intervening real part is certified by a whole symbol box or monotonicity and endpoints; the entire negative support and positive exterior are included. If a root stripe has width η and $|m'_M| < 100$, its symbol magnitude is at most 100η and its integrated weight in $d\xi/\pi$ is at most $100\eta^2/\pi$. Thus the omitted stripe payment is quadratic in its width. All such stripes remain paid in the continuous source.

The source panel lengths are at most $1/4$ in the head and 1 in ambient modules. Their parameter-2 ellipses have real halfwidth $5h/4$ and imaginary halfwidth $3h/4$. At every actual rectangle the complete holomorphic symbol is bounded by 2000 . Direct interval evaluation supplies this bound when sharp. A complete series majorant supplies a second admission: if $\Re w > 1/32$ and $|w - 1| < N = 200000$, then

$$|\Psi_0(w)| < 36 + \log N + \log(1 + 32N).$$

Indeed split $\Psi_0(w) = -\gamma + \sum_{k \geq 0} \{(k+1)^{-1} - (k+w)^{-1}\}$ at N . The first part is bounded by a harmonic sum and $32 + \log(1 + 32N)$; the complete remaining series costs at most 2 . The actual cosine terms then give the stated symbol cap.

Mapped Chebyshev arguments have modulus below 2 , hence $|T_j| \leq 5^j$. For two complex frequency variables the imaginary kernel argument is at most $3/4$. The complete theta envelope gives

$$|\hat{q}(z)| \leq \int_{\mathbb{R}} q(u) e^{|u|} du \leq \frac{16}{\pi} (\pi^2 + 2\pi + 2) e^{-\pi} < 6 \quad (|\Im z| \leq 1).$$

Both profile factors are included in each two-variable majorant. For an analytic profile bound M_{an} and total width L , safe Gaussian bounds are $64LM_{\text{an}}2^{-2n}$ for a metric and $128L^2M_{\text{an}}2^{-2n}$ for a mixed Gram. The mathematical quadrature concerns the true analytic integrands; the real node values then use the fixed kernel polynomial and its separately paid full model debit.

Source class	Observations	Gauss order	Profile bound	Entry pad
Exact-row head	46	256	10^{30}	10^{-100}
Ambient modules	240	224	10^{40}	10^{-80}

The head uses 614 negative and 1680 positive columns. Its maximal polynomial index is 16. Every ambient group retains its own literal block degrees, rational window and total integration width. The reader checks

$$\frac{2000 \cdot 5^{2j}}{\pi} < M_{\text{an}}, \quad \frac{12 \cdot 2000^2 \cdot 5^{2j}}{\pi^2} < M_{\text{an}}$$

for its actual maximal index j , then checks the Gaussian and quadratic root-stripe errors against the entry pads. The negative ridge is 10^{-29} throughout. The head positive ridge is 10^{-28} and the ambient positive ridges are 10^{-20} . Each complete metric error fits its stated ridge.

The typed index `data/m16/plan.json` binds all 206 groups to these literal objects. Positive source projections are contracted in their true frequency metrics. The complete negative return uses the interpolation bound (12), with $R = 2$ and the complete theta spatial tail; all 286 component masses are regenerated.

4.3 The original twelve-row head source

Write

$$\Xi(\xi) = \xi_{\text{Riemann}}(1/2 + i\xi) = \int_{\mathbb{R}} \psi(u) e^{-i\xi u} du.$$

Complete-theta integration by parts gives

$$\widehat{sz_{2j+1}}(\xi) = -i(-1)^j \xi^{2j+1} (\xi^2 + 1/4) \Xi(\xi).$$

For exact dyadic premixing coefficients A_{jk} , form

$$\tilde{z}_k = \sum_{j=0}^{10} A_{jk} z_{2j+1}, \quad P_k(\xi) = \sum_{j=0}^{10} A_{jk} (-1)^j \xi^{2j+1} (\xi^2 + 1/4)$$

before source evaluation. The actual mixed column is

$$\langle E_i, \tilde{z}_k \rangle = \int w(\xi) T_i(t_I(\xi)) P_k(\xi) \Xi(\xi) \frac{d\xi}{\pi}.$$

Exact combinations remain genuine nulls; their actual original Gram $A^* \Gamma_{11} A$ is retained. Appendix A gives the complete physical quadrature and reciprocal-theta bounds. The supplied complete enclosure differs from I in row-sum norm by less than $1.108 \cdot 10^{-60}$. This is a verified normalization, not a replacement of that Gram by I .

The polar row completes $\mathcal{Z} = [\tilde{z}_1, \dots, \tilde{z}_{11}, b]$. Its null crosses vanish exactly, and the complete Mellin identity gives

$$\|b\|_2^2 = \frac{1}{2} - \widehat{q}(0) > 0.$$

Thus Γ is the direct sum of the full eleven-row Gram and this positive scalar. On X , projection off these twelve rows equals $Q_{K_{11}}$. The row b implements a Hilbert constraint; the operator-null transport uses K_{11} alone.

The remaining polar mixed source is

$$\langle E_i, b \rangle = \int w(\xi) T_i(t_I(\xi)) B(\xi) \frac{d\xi}{\pi}, \quad B(\xi) = \int_{\mathbb{R}} \psi(u) \tanh(u/2) \sin(\xi u) du.$$

It has a separately funded odd degree-1536 polynomial on $[-500, 500]$ with real uniform error below 10^{-85} . Its complete construction uses ten physical panels on $[0, 5/2]$, order 192, eight explicit theta atoms and both full omitted tails. The physical integrand bound is $256e^{500 \cdot 3/32}$; its Gaussian payment is

$$2 \cdot 10 \cdot 8(1/8) \frac{256e^{500 \cdot 3/32} 2^{-384}}{1 - 1/2}.$$

The omitted atom and real-tail payments are respectively

$$10 \cdot 16\pi^2 9^4 e^{-81\pi}, \quad \frac{480e^{9U/2 - \pi e^{2U}}}{2\pi e^{2U} - 9/2}, \quad U = 5/2.$$

Their sum is below 10^{-90} at every frequency node. On the parameter-2 frequency ellipse a complete bound is $240 \cdot 189! / 3^{189}$. Lobatto aliasing costs at most four times this bound times 2^{-1536} ; the sum of all coefficient radii is also paid. The resulting total is below $5.790 \cdot 10^{-89}$. Its local degree-256 polynomial remainder is below $3.784 \cdot 10^{-121}$. This independent polar source funds the actual row, including its sine normalization.

For the true null and polar mixed integrands the complete contour bound $|\Xi(x+iy)| \leq 768e^{-|x|/2}$ and $|B(x+iy)| \leq 768$, $|y| \leq 2$, supplies source majorants. On each actual frequency ellipse, $|z| < 500$, $|T_i| < 5^{16}$ and $|m_{16}| < 10000$. The exact coefficient sum $B_k = \sum_{\ell} |[P_k]_{\ell}| 500^{\ell}$ bounds each premixed polynomial. For a total frequency length at most 1000, a safe Gaussian payment is

$$16 \cdot 1000 (10000 5^{16} 768 B_k / \pi) 2^{-512}.$$

On the real line, positivity of ψ gives $|\Xi(\xi)| \leq \Xi(0) < 1$. For at most 1000 root stripes of width at most η , their complete mixed-column payment is therefore at most $100 \cdot 1000 \eta^2 B_k / \pi$. The actual

stripe endpoints allow the smaller corresponding polynomial coefficient sums. The Gaussian and root-stripe errors are evaluated in each column, with all arithmetic balls retained. Complete row integration costs are below $8.281 \cdot 10^{-104}$ for quadrature and $2.522 \cdot 10^{-75}$ for stripes. Together with the full original Gram, the separate polar model and the complete ambient covariance, these are the joint inputs to (11). Exact row premixing does not import locality through the projection or through an inverse.

5 Complete raw arithmetic control and the original closure

Put $U_b = M_s S_b M_s$, $b = \log n$, $p_n = s(\frac{1}{2} \log n)^2$ and $q_n = s(\log n)^2$. The paired-shift estimate [1, 2] is

$$\|U_b + U_b^*\| \leq \sqrt{p_n^2 + 3s(0)^2 q_n}. \quad (17)$$

Lemma 5.1 (Whole arithmetic remainder). *For $N \geq 3$, let $A = 800\pi^2/99$ and*

$$\begin{aligned} r_N &= ((N+1)/N)^{5/2} e^{-\pi}, & t_N &= ((N+1)/N)^{5/2} e^{-\pi(2N+1)/2}, \\ F(N) &= \frac{AN^{3/2} \log N e^{-\pi N}}{1 - r_N} + \frac{A\sqrt{3e^{-\pi}} N^{3/2} \log N e^{-\pi N^2/2}}{1 - t_N}. \end{aligned}$$

Then $\|W_{>M}\| \leq F(M+1)$ for $M \geq 2$. For all $j \geq 1$,

$$\kappa_j = 2^{-j}, \quad M_j = \lceil j/2 \rceil + 10, \quad \delta_j := \|W_{>M_j}\| < 2^{-37} \kappa_j^2. \quad (18)$$

Proof. The complete theta envelope gives $s(u)^2 \leq A e^{4u - \pi e^{2u}}$ for $u \geq 0$. Apply (17), $\Lambda(n) \leq \log n$ and $\sqrt{x+y} \leq \sqrt{x} + \sqrt{y}$. The n th summand is at most

$$An^{3/2} \log n e^{-\pi n} + A\sqrt{3e^{-\pi}} n^{3/2} \log n e^{-\pi n^2/2}.$$

Since $\log x/x$ decreases for $x \geq 3$, the polynomial consecutive ratio is at most $((n+1)/n)^{5/2}$. The two decreasing geometric ratios give $F(N)$. For $N \geq 7$, the same ratios give $F(N+1) \leq F(N)/16$. The directed base admission $16F(12) < 2^{-37}$ and $k = \lceil j/2 \rceil$ imply $F(k+11) \leq F(12)16^{-(k-1)}$. Now $\kappa_{2k}^2 = 16^{-k}$ and $\kappa_{2k-1}^2 = 4 \cdot 16^{-k}$, giving (18). $\square$

Lemma 5.2 (Common-row cutoff-sixteen payment). *The complete remainder satisfies*

$$\|W_{>16}\| < 7.86 \cdot 10^{-20} + F(36) < 7.860001 \cdot 10^{-20}, \quad F(36) < 4.937957118422 \cdot 10^{-45}.$$

Proof. The omitted prime powers below 36, with their respective prime bases, are

$$\begin{array}{c|cccccccc} n & 17 & 19 & 23 & 25 & 27 & 29 & 31 & 32 \\ \text{base} & 17 & 19 & 23 & 5 & 3 & 29 & 31 & 2. \end{array}$$

For their finite symmetric sum, the inequality $2|xy| \leq |x|^2 + |y|^2$ and translation give the common Schur row

$$R_{16}(u) = s(u) \sum_{16 < n < 36} c_n \{s(u + \log n) + s(u - \log n)\}.$$

Its supremum bounds the norm of this complete paired sum.

For $v \geq 0$ put $r = \pi e^{2v}$. The exact theta formula is

$$s(v)^2 = \frac{2}{1 + e^{-v}} \sum_{k \geq 1} (4r^2 k^4 - 6rk^2) e^{-rk^2}.$$

All atoms are positive. For $k \geq 2$ their polynomial majorants have successive ratio at most $(3/2)^4 e^{-5\pi} < 1/2$. The whole tail from $k = 2$ is bounded by $128r^2 e^{-4r}$. Since $4r^2 - 6r \geq 2r^2$, this gives

$$s(v)^2 \leq \frac{2}{1 + e^{-v}} (4r^2 - 6r) e^{-r} (1 + 64e^{-3r}).$$

On each outward interval for $|v|$, evaluate the increasing polynomial and prefactor at the upper endpoint, and decreasing exponentials at the lower r endpoint. The standalone reader recursively covers $[0, 4]$ by 740 exact dyadic cells, each with $R_{16} < 7.86 \cdot 10^{-20}$; rational endpoint adjacency verifies the union. For $u \geq 4$, the complete envelope $s(u) \leq 13e^{2u - \pi e^{2u}/2}$ decreases, $s < 3$, and each $c_n < 1$. Thus

$$R_{16}(u) \leq 48 \cdot 13e^{8 - \pi e^8/2} < 10^{-1000}.$$

Evenness covers the other arm. Lemma 5.1 pays every remaining prime power by $F(36)$, proving the stated full-line bound. $\square$

Proposition 5.3 (Paired closure). *Identity (5) and bounded receiving returns extend from the exact-polar smooth core to its original completion V .*

Proof. The weighted shift pairs converge in norm by Lemma 5.1. The differences $m_M - a$ and $p_M - a$ are bounded. Core expressions therefore differ from the positive graph form by bounded forms. Graph convergence passes the identities and inequalities to V , with the paired realization specified in Section 2. $\square$

6 The complete cutoff-sixteen certificate

The exhaustive actual sign cover has 571 simple roots in strict 220-bit rational brackets and 286 negative cover intervals. Every intervening sign cell is certified by an interval box or a monotone derivative and its endpoints. The finite partition has 120125 cells; the complete positive exterior starts at 262144. The root count, adjacency, sign and whole exterior are replayed together.

The allocation consists of a head and 205 ambient modules.

Collection	Observations	Windows	Negative columns	Positive columns
Exact-row head	46	1	614	1680
Ambient modules	240	205	2622	19198
Complete allocation	286	206	3236	20878

The actual rational windows have disjoint interiors. Every negative interval is assigned once. Source-space covariance across different groups is handled by adding their whole-space form inequalities on the same vector; frequency ownership gives the positive resource bound.

6.1 Complete head covariance and an exact congruence

The head owns $[0, 210]$. Its 105 width-2 positive bins each have 16 columns. The negative and positive frequency ridges are 10^{-29} and 10^{-28} . The full 2294×2294 ambient Gram is regenerated from 151 source moment blocks. Every one of its 2294×12 mixed row entries uses the actual symbol m_{16} . The complete physical row covariance Γ is retained and inverted as an enclosure; the exact row formula (10) is used directly. The normalized complete kernel debit is

$$\delta < 4.146028452567 \cdot 10^{-58}.$$

A proposed inverse action at $\kappa_0 = 8.5 \cdot 10^{-19}$ supplies an exact 1680×614 dyadic F with at most 512 mantissa bits. Only its literal entries are used in the robust target. An exactly upper-triangular dyadic $\mathcal{R}$ has positive diagonal; the full enclosing matrix $\mathcal{R}^*S(\kappa_0)\mathcal{R}$ has 614 strict Gershgorin rows, with lower bounds exceeding 0.99.

For this same fixed F , put $D_F = F^*D_pF$. Every source and row payment stays fixed in the affine identity

$$S(\kappa) = S(\kappa_0) + (\kappa - \kappa_0)(D_n - D_F).$$

At $\kappa_{\text{head}} = 4.5 \cdot 10^{-19}$ all 614 directed pivots of the complete transformed target $\mathcal{R}^*S(\kappa_{\text{head}})\mathcal{R}$ are positive. The complete recorded pivot floor exceeds 0.25750119. An independent row-recursive LDL evaluation of this same enclosing target agrees with the strict sign. The full continuous head approximation costs less than $1.176886 \cdot 10^{-26}$. Thus, on the original head recipient,

$$N_{\text{head}} \leq P_{[0,210]} + (4.5 \cdot 10^{-19} + 1.176886 \cdot 10^{-26})I.$$

The exact source-aware congruence controls the full matrix, including directions whose unconditioned interval elimination is poorly scaled.

6.2 Ambient sources and the full original-domain return

Each of the other 205 groups uses $\kappa_g = 10^{-24}$ and the actual source metrics, complete mixed covariance and an exact dyadic F_g . All 2622 negative directions pass. The complete ambient fee is below $2.050383338305 \cdot 10^{-22}$. The full negative interpolation, spatial-tail and ridge charge is

$$\tau < 5.010268429993 \cdot 10^{-26}.$$

The head and ambient parts are included exactly once in this number.

Proof of Theorem 1.1. For $h \in V$ put $u = Q_{K_{11}}h$. Lemma 2.1 gives $u \in V \cap K_{11}^\perp$, $q_H[u] = q_H[h]$ and $\|u\|_2 \leq \|h\|_2$. All receiving comparisons apply to this same u . The complete negative cover and disjoint positive windows give

$$N_{16}[u] \leq P_{16}[u] + \{4.5 \cdot 10^{-19} + 205 \cdot 10^{-24} + \tau\}\|u\|_2^2.$$

Using (5) and Lemma 5.2,

$$q_H[h] \geq -\{4.5 \cdot 10^{-19} + 205 \cdot 10^{-24} + \tau + 7.86 \cdot 10^{-20} + F(36)\}\|h\|_2^2.$$

Outward arithmetic gives a paid sum below $5.288050501026843 \cdot 10^{-19} < 5.289 \cdot 10^{-19}$. Proposition 5.3 supplies the original minimal-domain return. $\square$

7 Growing-cutoff constraint costs

The source-specific input is the genuine complete-null family (6), with its original-domain and operator-null identities established in Section 2. Appendix B funds the explicit original graph-norm bound used here, giving

$$\|z_{2i+1}\|_2 \leq 2^{30}256^i(2i+1)!. \quad (19)$$

Lemma 7.1 (Complete low-frequency mass). *For $M \geq 16$, throughout $0 \leq \xi \leq \varepsilon_M = 1/(2 \log M)$,*

$$n_M(\xi) \geq \frac{\log 2}{2}\sqrt{M}, \quad \hat{\psi}(\xi) > \frac{1}{4}.$$

Proof. Write $\vartheta_C(M) = \sum_{n \leq M} \Lambda(n)$. For $n = \lfloor M/2 \rfloor$ the central binomial coefficient divides $\text{lcm}(1, \dots, 2n)$: each prime valuation is a sum of at most $\lfloor \log_p(2n) \rfloor$ terms equal to zero or one. The largest binomial coefficient is at least $2^{2n}/(2n+1)$, so

$$\vartheta_C(M) \geq (M-1) \log 2 - \log(M+1).$$

Consequently $S_M = \sum_{2 \leq n \leq M} \Lambda(n)/\sqrt{n} \geq (\log 2/2)\sqrt{M}$ for $M \geq 16$. On this interval $\cos(\xi \log n) \geq 7/8$. The digamma series gives $a(\xi) \leq 17\xi^2$, and $17/(4 \log^2 M) \leq (3 \log 2/8)\sqrt{M}$ holds from $M = 16$ onward. Hence $m_M(\xi) \leq 17\xi^2 - (7/4)S_M \leq -S_M$.

The complete Mellin identity gives $\int_{\mathbb{R}} \psi = \xi_{\text{Riemann}}(1/2) > 3/8$. The complete atom envelope, substitution $r = \pi e^{2u}$ and $\log^2(r/\pi) \leq r^2/\pi^2$ give

$$\int_{\mathbb{R}} u^2 \psi(u) du \leq \frac{2400}{99} \pi^{-9/4} < 3.$$

Here the remaining gamma integral has power $13/4$ and is bounded by $\int_0^\infty r^4 e^{-r} dr = 4!$. Thus $\cos t \geq 1 - t^2/2$ implies $\widehat{\psi}(\xi) \geq 3/8 - (3/2)\xi^2 > 1/4$ on $|\xi| \leq 1/4$. As $\varepsilon_M < 1/4$, the claim follows. $\square$

For $d \geq 0$ define

$$\begin{aligned} g_d &= \frac{3(d!)^2}{(d+1)(4d+3)^{2d+2}}, & L_d &= 2^{60} \sum_{i=0}^d 256^{2i} ((2i+1)!)^2, \\ c_d &= \frac{(\log 2)g_d}{512\pi 2^{4d+3}L_d}, & b_d(M) &= \frac{c_d \sqrt{M}}{(\log M)^{4d+3}}. \end{aligned} \quad (20)$$

Lemma 7.2 (Moment floor and null observation growth). *For $H_d = (1/(2i+2k+3))_{0 \leq i,k \leq d}$, $\lambda_{\min}(H_d) \geq g_d$. Every unit z in $Z_d = \text{span}(z_1, z_3, \dots, z_{2d+1})$ has $N_M[z] \geq b_d(M)$ for $M \geq 16$.*

Proof. The exact Cauchy inverse formula bounds each entry of H_d^{-1} by

$$\frac{(4d+3)^{2d+2}}{3 \cdot 2^{2d} i! (d-i)! k! (d-k)!} \leq \frac{(4d+3)^{2d+2}}{3(d!)^2}.$$

The denominator contains $2i+2k+3 \geq 3$ and the two Cauchy-node difference products; the second inequality uses binomial coefficients at most 2^d each. The row-sum norm gives the eigenvalue floor. For $z = \sum_{i=0}^d a_i z_{2i+1}$,

$$\widehat{sz}(\xi) = -i\xi(\xi^2 + 1/4)\widehat{\psi}(\xi) \sum_{i=0}^d (-1)^i a_i \xi^{2i}.$$

Lemma 7.1 bounds its squared modulus below by $\xi^2 |\sum (-1)^i a_i \xi^{2i}|^2 / 256$ on $[0, \varepsilon_M]$. Scaling $\xi = \varepsilon_M x$ gives moment floor $\varepsilon_M^{4d+3} g_d \sum |a_i|^2$. By (19), $\|z\|_2^2 \leq L_d \sum |a_i|^2$, so unit vectors have coefficient mass at least $1/L_d$. Multiplying the negative mass bound and measure $1/\pi$ gives $b_d(M)$. $\square$

Let $X_M \subset X$ be any closed subspace of additional codimension at most d , with the original form restriction. Suppose $0 \leq N_M - N_{\star, M} \leq \tau_M I$ on it and define

$$\Lambda_M = \|(P_M + \kappa_M I)^{-1/2} N_{\star, M} (P_M + \kappa_M I)^{-1/2}\|, \quad \kappa_M > 0. \quad (21)$$

The inverse refers to the associated positive form on this constrained space.

Theorem 7.3 (Arbitrary-constraint inverse-margin obstruction). *With $\delta_M = \|W_{>M}\|$,*

$$\Lambda_M \geq 1 - \frac{\kappa_M + \delta_M + \tau_M}{b_d(M) + \kappa_M + \delta_M}, \quad M \geq 16. \quad (22)$$

For fixed d , bounded κ_M, τ_M and $\delta_M \rightarrow 0$, this implies $\liminf_{M \rightarrow \infty} \Lambda_M \geq 1$.

Proof. The $(d+1)$ -dimensional genuine-null space Z_d intersects X_M nontrivially even when X_M varies. Choose a unit z in the intersection; it lies in V and satisfies $q_H[z] = 0$. Put $a = N_M[z] \geq b_d(M)$. Equation (5) gives

$$(P_M + \kappa_M I)[z] \leq a + \delta_M + \kappa_M, \quad N_{\star, M}[z] \geq a - \tau_M.$$

If $a - \tau_M \geq 0$, the generalized Rayleigh quotient is at least $(a - \tau_M)/(a + \delta_M + \kappa_M)$. This expression increases with a . If $b_d(M) - \tau_M < 0$, the advertised bound is negative and follows instead from $\Lambda_M \geq 0$. Otherwise $a \geq b_d(M) \geq \tau_M$, so the first case applies and proves (22). For fixed d , $b_d(M) \rightarrow \infty$. $\square$

This excludes a fixed strict cap $1 - \gamma$, $\gamma > 0$, under bounded constraint count, without assuming an upper bound for Λ_M .

Theorem 7.4 (Quantitative growing-row cost). *Use (18) and an actual approximation with $\tau_j \leq \kappa_j^2$. If $j \geq 2^{24}$ and additional codimension $D_j \leq j/(64 \log j)$, then*

$$\Lambda_j \geq 1 - 2e^{-(\log 2 - 1/8)j}.$$

Any fixed strict cap $\Lambda_j \leq 1 - \gamma$ for all sufficiently large j therefore requires $D_j > j/(64 \log j)$ eventually.

Proof. The factorial formulas in (20) imply

$$c_d \geq \exp[-4d \log(d+1) - 20d - 6 \log(d+1) - 60], \quad d \geq 0.$$

For $d \geq 1$ use $d!/(2d+1)! \geq (2d+1)^{-(d+1)}$, $d+1 \leq 2d$, $2d+1 \leq 3d$, $4d+3 \leq 7d$; the remaining coefficients satisfy $20 \log 2 + 2 \log 21 < 20$ and $72 \log 2 + \log 4 + 2 \log 21 - \log(3 \log 2/\pi) < 60$. The case $d = 0$ is direct. Put $t = \log j \geq 16$. Since $M_j \leq j$ and $D_j + 1 \leq j$,

$$\log b_{D_j}(M_j) \geq -4D_j \log t - 3 \log t - 4D_j t - 20D_j - 6t - 60.$$

Terms involving D_j cost at most $j/16 + j(\log t + 5)/(16t) \leq 25j/256$. The remaining cost is at most $11t \leq 7j/256$ for $t \geq 16$. Thus $b_{D_j}(M_j) \geq e^{-j/8}$. The paid schedule and $\tau_j \leq \kappa_j^2$ give $\kappa_j + \delta_j + \tau_j < 2\kappa_j$. Apply (22). Its lower bound approaches one, proving the eventual dimension requirement. $\square$

The order is $\log(1/\kappa)/\log \log(1/\kappa)$ along this schedule. It is a necessary constraint count for a fixed relative margin. Architectures that explicitly budget shrinking margins are also compatible with the theorem.

8 Original and receiving Gram floors

Theorem 8.1 (Full original null Gram). *With $\beta_d = 2^{-8d-20}g_d$, the complete original Hilbert Gram Γ_d of (6) satisfies*

$$\beta_d I \leq \Gamma_d \leq L_d I.$$

For $D_\alpha = \alpha + s^2/5$ and $y_i = D_\alpha^{-1/2} z_{2i+1}$, the complete row Gram obeys

$$\frac{\beta_d}{\alpha + 16/5} I \leq G_{\alpha,d} \leq \frac{L_d}{\alpha} I.$$

Proof. The complete atom envelope gives $s(u)^2 \leq 16\pi^2 e^{4|u| - \pi e^{2|u|}} < 16$. For $z = \sum a_i z_{2i+1}$ and $f = sz$, Plancherel and the moment floor imply

$$\begin{aligned} \|z\|_2^2 &\geq \frac{1}{16} \|f\|_2^2 \geq \frac{1}{4096\pi} \int_0^{1/4} \xi^2 \left| \sum (-1)^i a_i \xi^{2i} \right|^2 d\xi \\ &\geq \frac{g_d}{4096\pi 4^{4d+3}} \sum |a_i|^2 \geq \beta_d \sum |a_i|^2. \end{aligned}$$

The upper bound is (19). Finally $\alpha \leq D_\alpha \leq \alpha + 16/5$, proving the receiving comparison. $\square$

Proposition 8.2 (Exact coordinates and precision cost). *Suppose a positive continuous r -row Gram has $\ell I \leq G \leq UI$, and a rational symmetric G_0 satisfies $\|G_0 - G\| \leq \ell/16$. There is an effectively constructible invertible dyadic S with*

$$\frac{3}{4}I < S^*GS < \frac{5}{4}I, \quad \text{cond}(S^*GS) < 5/3.$$

For the receiving Gram in Theorem 8.1, $0 < \alpha \leq 1$, the required precision is $O(\log(1/\alpha) + d \log(d+2))$ bits. For the original Gram omit the first term. At $\alpha_j = 2^{-j}$ and $d_j = O(j/\log(j+2))$ it is $O(j)$.

Proof. An exact inverse Cholesky factor B_0 of G_0 gives $14I/15 \leq B_0^*GB_0 \leq 16I/15$. Approximate each entry by a dyadic within 2^{-b} , with $2^{-b} \leq 1/(32r\sqrt{U})$, obtaining S . Then $\|G^{1/2}(S - B_0)\| \leq 1/32$. Squared singular values lie between $(\sqrt{14/15} - 1/32)^2 > 3/4$ and $(\sqrt{16/15} + 1/32)^2 < 5/4$. Positive rational Cholesky with interval refinement makes the construction effective. The factorial formulas imply $\log \beta_d^{-1}, \log L_d = O(d \log(d+2))$. They bound b , integer parts via $\|B_0\| \leq 2/\sqrt{\ell}$, and absolute Gram accuracy after accounting for its magnitude U . This is a precision statement rather than an operation-count estimate. $\square$

Exact dyadic combinations are formed before source profiles are evaluated. Transforming rounded raw samples requires a separate amplified-error payment.

8.1 A compatible resolvent recipient

The same nulls apply to the constrained archimedean form $A[h] = \int a(\xi) |\widehat{sh}(\xi)|^2 d\xi / (2\pi)$. Write $H = A + B$. The complete shift estimate gives $\|B\| < 256$: use $|m_0| < 6$, $\|s^2\|_\infty < 16$, $\sum_{n \leq 11} c_n < 4.261$, and the paid raw tail. For $C = A + \epsilon I$, spectral Cauchy-Schwarz on a genuine null z gives

$$\|z\|^4 \leq C[z] \langle z, C^{-1}z \rangle \leq (256 + \epsilon) \|z\|^2 \langle z, C^{-1}z \rangle.$$

The complete resolvent-row Gram therefore satisfies

$$\frac{\beta_d}{256 + \epsilon} I \leq Z_d^* C^{-1} Z_d \leq \frac{L_d}{\epsilon} I.$$

Proposition 8.2 applies with these bounds. The inverse is that of the associated constrained form, without a compression-inverse or null-invariance identification. This is the original resolvent geometry studied in [5].

9 Reproducibility and mathematical use

The typed `data/m16/plan.json` contains the complete 286-observation allocation and its 206 positive windows. The complete reader `code/verify_complete.py` first checks every physical row-entry enclosure and then runs the whole-certificate `code/reproduce_m16.py --mode all` reconstruction. It reconstructs the complete mixed source Grams from compact actual moments, the original twelve-row shorting, the exact dyadic receiving maps and the full directed matrix signs. Its optimized counterpart retains every explicit acceptance guard. `REPRODUCING.md` specifies dependencies, resource use, fresh source generation and whole-certificate versus selected-group replay scopes.

The full continuous kernel, source, root, Gaussian, approximation, frequency metric and arithmetic-tail payments are part of this return. The true original row Gram is kept as an enclosure. The reader also checks complete frequency ownership and scalar admissions for the theoretical resource estimates. Checksums identify the supplied data; the displayed reductions and directed signs establish the certificate. Interval arithmetic uses `python-flint/Arb` [9].

The fixed-cutoff result combines exact-null transport, complete receiving and an explicit infinite-dimensional return. The growing-cutoff estimates quantify the constraint and coordinate resources of fixed strict relative margins on the same original realization. These conclusions retain their own recipients and approximation payments.

A Complete physical source for the original null Gram

The covariance in Section 4.3 uses the original norm, including its reciprocal-theta factor. Here we give the bounds underlying its physical quadrature. Set

$$F_j = (D^2 - 1/4)D^{2j+1}\psi, \quad G_{jk}^0 = 2 \int_0^\infty \frac{F_j(u)F_k(u) \cosh(u/2)}{\psi(u)} du, \quad 0 \leq j, k \leq 10.$$

Define the exact polynomial operator $\mathcal{D}Q = \frac{1}{2}Q + 2rQ' - 2rQ$ and

$$Q_j = (\mathcal{D}^2 - 1/4)\mathcal{D}^{2j+1}(4r^2 - 6r), \quad d_j = \deg Q_j = 2j + 5, \quad S_j = \sum_\ell |[Q_j]_\ell|.$$

Thus $F_j(z) = e^{z/2} \sum_{n \geq 1} Q_j(\pi n^2 e^{2z}) e^{-\pi n^2 e^{2z}}$. All these polynomials have exact rational coefficients.

Consider $z = x + iy$, $x \geq 0$, $|y| \leq .01$, and put $r = \pi e^{2x}$ and $r_c = \pi e^{2z}$. Then $\Re r_c > .99r$ and $|r_c| = r \geq \pi$. The first theta atom satisfies $|4r_c^2 - 6r_c| \geq 2r^2$. Relative to this atom, the sum of the remaining absolute values is at most

$$3 \sum_{n \geq 2} n^4 e^{-.99r(n^2-1)} \leq 64e^{-3.99r} < .1.$$

Indeed the successive-term ratio is at most $(3/2)^4 e^{-5.99\pi} < 1/4$. Consequently the complete denominator is nonzero and

$$|\psi(z)| \geq \frac{9}{5} r^2 e^{x/2-r}. \quad (23)$$

Splitting $.99 = .24 + .75$ and maximizing $t^{d_j} e^{-.24t}$ gives

$$C_j = \frac{S_j(d_j/.24)^{d_j} e^{-d_j}}{1 - e^{-3\pi/4}}, \quad |F_j(z)| \leq C_j e^{x/2-3r/4}.$$

Since $|\cosh(z/2)| \leq e^{x/2}$, it follows that

$$\left| \frac{F_j(z)F_k(z)\cosh(z/2)}{\psi(z)} \right| \leq \frac{5C_jC_k}{9\sqrt{\pi}} r^{-3/2} e^{-r/2} \leq M_{jk} := \frac{5C_jC_k}{9\pi^2} e^{-\pi/2}. \quad (24)$$

Each F_j is odd and ψ is even, so the Gram integrand is even. This also funds the part of the first quadrature ellipse with negative real part by reflection to $|\Re z|$.

The finite interval $[0, U]$, $U = 3$, is split into 120 equal cells with halfwidth $h = 3/240$. For the Bernstein parameter $\rho = 7/5$, $h(\rho - \rho^{-1})/2 < .01$. Order 512 Gaussian quadrature, including both physical arms, therefore has complete error at most

$$8UM_{jk} \frac{\rho^{-1024}}{1 - \rho^{-1}}.$$

At every real node eight theta atoms are evaluated. The remaining denominator and jet tails are respectively bounded by

$$8e^{u/2}r^29^4e^{-81r}, \quad 2S_j e^{u/2}r^{d_j}9^{2d_j}e^{-81r}.$$

The successive-term bounds $(10/9)^{2d_j}e^{-19\pi} < 1/2$ fund all omitted atoms. These positive-radius balls are included before the reciprocal of ψ is taken; positivity of the complete real denominator is checked. Finally $r(U) = \pi e^6 > 1200$. Integrating the first bound in (24), with $du = dr/(2r)$, funds the two-arm physical tail by

$$\frac{10C_jC_k}{9\sqrt{\pi}} 1200^{-5/2}e^{-600}.$$

Both errors are added to every raw Gram entry. The exact dyadic map A then gives the complete enclosure A^*G^0A , with all interval radii retained. The row-sum allowance stated in Section 4.3 and the complete inverse-enclosure tests in Section 6 use this enclosure. The portable physical generator evaluates these formulas anew; it does not substitute an identity matrix for the conditioned Gram.

B An explicit original graph budget for all null jets

The finite-jet null and minimal-domain input is inherited from [4]. We supply the quantitative graph bound required by (19). Put $q_0 = (D^2 - 1/4)\psi$ and $U(t, u) = q_0(u + t)/s(u)$ for $|t| \leq 1/8$. For $u \geq 0$ let $r = \pi e^{2u}$. The complete atom polynomials are

$$\begin{aligned} Q_0(x) &= 16x^4 - 112x^3 + 164x^2 - 36x, \\ Q_1(x) &= -32x^5 + 360x^4 - 1056x^3 + 810x^2 - 90x. \end{aligned}$$

For $z = u + t$, the series for $q_0(z)$ is $e^{z/2} \sum_{n \geq 1} Q_0(\pi n^2 e^{2z}) e^{-\pi n^2 e^{2z}}$; Q_1 gives q'_0 . On the whole parameter disk,

$$\Re(\pi n^2 e^{2(u+t)}) \geq e^{-1/4} \cos(1/4) r n^2 > \frac{3}{4} r n^2.$$

The absolute polynomial coefficient sums are 328 and 2348, and the arguments have modulus greater than one. Complete ratio bounds give

$$\sum_{n \geq 1} n^8 e^{-3rn^2/4} \leq \frac{3}{2} e^{-3r/4}, \quad \sum_{n \geq 1} n^{10} e^{-3rn^2/4} \leq 4e^{-3r/4}.$$

For the first, every consecutive ratio is at most $256e^{-27/4} < 1/3$. For the second, the second term relative to the first is at most $1024e^{-27/4} < 3/2$, and subsequent ratios are at most $(3/2)^{10}e^{-45/4} < 1/2$. Using $e^{17/16} < 3$ and $e^{21/16} < 4$, this yields

$$|q_0(u+t)| \leq 1500r^4e^{u/2-3r/4}, \quad |q'_0(u+t)| \leq 40000r^5e^{u/2-3r/4}.$$

Evenness by theta inversion treats the other real arm.

The first positive real atom gives $\psi(u) \geq 2r^2e^{u/2-r}$ and $1/s(u)^2 \leq e^r/(2r^2)$. Substitute $du = dr/(2r)$, use even majorants and enlarge integer-power gamma integrals to $(0, \infty)$. Then

$$\begin{aligned} \|U(t)\|_2^2 &\leq (1500^2/2) \int_{\pi}^{\infty} r^{11/2}e^{-r/2} dr \leq 103680000000, \\ \|q'_0(\cdot+t)\|_2^2 &\leq 40000^2 \int_{\pi}^{\infty} r^{19/2}e^{-3r/2} dr \leq 40000^2 10!(2/3)^{11}. \end{aligned}$$

The digamma series gives $0 \leq a(\xi) \leq 17\xi^2$: the zero-th term contributes 16, and all later terms contribute at most $(1/4) \sum_{k \geq 1} k^{-3} \leq 3/8$. Plancherel therefore implies

$$\|U(t)\|_{\text{graph}}^2 \leq 103680000000 + 17 \cdot 40000^2 10!(2/3)^{11} < 2^{52}.$$

These uniform complete-tail envelopes also prove Hilbert-valued holomorphy by dominated integration. Let $T(t) = P_{\text{odd}}U(t)$. Evenness of s, a makes P_{odd} contract the graph norm. The Taylor coefficients of T are $z_{2i+1}/(2i+1)!$ at odd orders. They lie in the original V by the inherited finite-jet theorem. Since V is closed in this graph space, the Taylor sums and their limits lie in V . Their complete operator-null limits remain in $\ker H$, which is closed in the original Hilbert space.

The removable even-parameter family

$$\mathcal{A}(w) = \frac{q_0(\cdot + \sqrt{w}) - q_0(\cdot - \sqrt{w})}{2\sqrt{w}s} = \sum_{i \geq 0} \frac{w^i z_{2i+1}}{(2i+1)!}$$

is independent of the square-root choice and holomorphic for $|w| < 1/64$. Cauchy's estimate on the t -circle of radius $1/8$ gives $\|T(t)/t\|_{\text{graph}} \leq 2^{30}$ for $|t| \leq 1/16$. A second Cauchy estimate on $|w| = 1/256$ gives

$$\|z_{2i+1}\|_{\text{graph}} \leq 2^{30} 256^i (2i+1)!,$$

which includes (19). All approximations retain the paired original graph topology.

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