

RH, Approximately: Certified Lower Bounds for Odd Weil Positivity

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Luminary Oddities

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Abstract

We prove the complete lower bound $q_H[h] \geq -1.5 \cdot 10^{-11} \|h\|^2$ on the original constrained minimal odd domain in its reciprocal- θ norm. The certificate retains the full joint covariance of 4080 phase columns and five exact polar/null rows. Shorting that certified 4085-dimensional receiving Gram gives a directed 4080-dimensional exact-row Schur certificate. Its proof includes the complete arithmetic remainder, infinite frequency and physical tails, source conversion errors and the original-domain return. A real Bessel suppression argument and complete spatial-endpoint control sharpen the approximation without changing the frame. Three complementary compact-support certificates, in the distinct ordinary physical L^2 norm, give $A_8 \succeq 10^{-40}I + 4.75 \cdot 10^{-11}P_{e7,8}$, $A_9 \succeq 10^{-50}I + 4 \cdot 10^{-12}P_{e8,9}$ and $A_{10} \succeq 10^{-60}I + 10^{-13}P_{e9,10}$, with explicit sharp directional upper trials and joint eleven-, thirteen- and fifteen-feature receivers. An exact ground transformation preserves signed arithmetic pairing through closure even where the two positive edge energies separately diverge. It yields a norm-convergent paired tail, a sharper localized sign bound and exact compatibility with polar repair. Further results give fixed-null-prefix funding, all-support γ approximation, whole complementary-space bounds and an explicit fixed-cutoff exterior obstruction. These are certified original-domain and physical receiving results with their respective norms and hypotheses.

1 Two norms and one complete physical form

Use $\hat{f}(\xi) = \int_{\mathbb{R}} e^{-iu\xi} f(u) du$ and Fourier measure $d\xi/(2\pi)$. Let

$$\begin{aligned} \mathcal{M}f(z) &= \int_{\mathbb{R}} e^{zu} f(u) du, \\ \psi(u) &= \sum_{n \geq 1} e^{u/2} (4r_n^2 - 6r_n) e^{-r_n}, \quad r_n = \pi n^2 e^{2u}, \\ c(u) &= \cosh(u/2), \quad s(u) = \sqrt{\psi(u)/c(u)}. \end{aligned}$$

Theta inversion makes ψ positive and even, and $M \psi(z) = \xi(1/2+z)$. Define the physical form

$$\begin{aligned} Q[p] &= \int_{\mathbb{R}} [\Re \Psi_0(1/4 + i\xi/2) - \log \pi] |\hat{p}(\xi)|^2 \frac{d\xi}{2\pi} \\ &\quad - \sum_{n \geq 2} c_n \langle p, (S_{\log n} + S_{-\log n}) p \rangle, \\ c_n &= \Lambda(n)/\sqrt{n}, \quad S_b p(u) = p(u+b). \end{aligned}$$

For odd p , put

$$m(p) = \int_{\mathbb{R}} \sinh(u/2) p(u) du, \quad \mathcal{W}[p] = Q[p] - 2|m(p)|^2.$$

This is the complete physical odd form, retaining the negative pole coefficient 2 and every prime power. Define $p = sh$. Its original reciprocal- θ norm is

$$\|h\|^2 = \int_{\mathbb{R}} |p(u)|^2 \frac{\cosh(u/2)}{\psi(u)} du.$$

Put $b_+ = \sqrt{\psi c} = sc$, $b_- = \sqrt{\psi c} \tanh(u/2) = s \sinh(u/2)$, where $c(u) = \cosh(u/2)$, and define

$$\mathcal{K} = \{h \in L^2(\mathbb{R}) : \langle h, b_+ \rangle = \langle h, b_- \rangle = 0\}, \quad a(\xi) = \Re \Psi_0(1/4 + i\xi/2) - \Psi_0(1/4) \geq 0.$$

For odd vectors the first row vanishes automatically. The original minimal odd domain V is the closure of $C_{c,\text{odd}}^\infty(\mathbb{R}) \cap \mathcal{K}$ in the form norm

$$\|h\|_{\text{form}}^2 = \|h\|^2 + \int_{\mathbb{R}} a(\xi) |\widehat{sh}(\xi)|^2 \frac{d\xi}{2\pi}.$$

The closed form is $q_H[h] = Q[sh] = \mathcal{W}[sh]$ on V , because $m(sh) = 0$ there. Its complete weighted arithmetic shifts are bounded; all closures below use this specified minimal domain.

Principal original-domain result. On the original minimal odd domain V , with exactly the norm just defined,

$$q_H[h] \geq -\frac{3}{200000000000} \|h\|^2 = -1.5 \cdot 10^{-11} \|h\|^2, \quad h \in V.$$

Theorem 1 is stated in Section 2, where full covariance and exact-row shorting reduce the receiving comparison. Section 3 proves the complete original-domain bound. Sections 4 and 5 develop the complementary physical and paired-tail mechanisms. Their ordinary compact-support L^2 floors are not reinterpreted as original weighted floors.

The localized operators A_Y instead act on odd physical $p \in L^2(-a_Y, a_Y)$, where $a_Y = \frac{1}{2} \log Y$, with form $\mathcal{W}[p]$. For an original-domain vector with $p = sh$ supported there, a physical bound $A_Y \succeq \varepsilon I + \mu P_e$ gives the exact transport

$$q_H[h] \geq \varepsilon \|sh\|_{L^2}^2 + \mu |\langle se, h \rangle|^2.$$

The scalar floor is expressed in the physical norm $\|sh\|_{L^2}$; the two norms remain distinct.

The physical estimates complement existing localized positivity results. Liu's Theorem B treats half-width $17/16$, and the teal-sea repository reports an odd certificate at 1.19. Lihongmin's complete-form certificate reaches $(\log 11)/2$ in both parities, with the negative odd pole retained. That theorem supplies positivity on a wider physical window without printing an L^2 spectral floor; an elimination pivot is not such a floor. The results here specify directional lower operators and matched joint receivers, alongside a global bound in the distinct reciprocal- θ norm.

2 The complete lower bound and exact-row receiving

Theorem 1 (complete original-domain lower bounds). For the original constrained minimal odd domain V of Section 1,

$$q_H[h] \geq -\frac{3}{200000000000} \|h\|^2 = -1.5 \cdot 10^{-11} \|h\|^2.$$

The target is formed from the complete **4085-dimensional joint receiving Gram**, retaining all five exact polar/null rows before shorting to a **4080-dimensional phase target**. Its directed column strips retain every entry and every off-diagonal row sum. The original norm and minimal domain are the same as in the previously published $1.88 \cdot 10^{-9}$ bound; the improvement factor is $376/3$.

The principal estimate uses a bounded receiving comparison in the original Hilbert metric. Its complete signed form remains a quadratic form on the transformed original domain. We first state the finite-to-infinite receiving mechanism, and then verify every ingredient and the original-domain return in Section 3.

Use $\alpha = 10^{-11}$ and $\mu = 1/5$, and put $t_\alpha = s/\sqrt{\alpha + \mu s^2}$. The full map $W = [W_p, W_r]$ has 4080 low-frequency phase columns and five transformed exact polar/null rows. Let $G = W^*W$, let G_0 be its dyadic source center, and set $D_0 = \text{diag}(2I_{4080}, I_5)$. The bounded positive exterior is $0 \preceq O \preceq \tau I$; its covariance is $T_{\text{cov}} = W^*OW$. Write $C_{0,pp}$ for the raw odd-folded dyadic signed phase-coefficient center in the $D_{\text{phase}} = 2I$ normalization, and A_{true} for the exact phase-coefficient operator in $W_p A_{\text{true}} W_p^*$. Its source payment gives $A_{\text{true}} \preceq (C_{0,pp} + 2e_C I)/4$. The source and approximation payments are established in Section 3 and Appendices A and B.

Let $G = W^*W$ and $T_{\text{cov}} = W^*OW$. The actual dyadic Fourier matrix has all $352 \cdot 4085 = 1437920$ squared entries included in its directed Frobenius sum, giving $\|F\|_F < 5.374924$. Its actual maximal conversion error is below 2^{-57} . Thus $\delta_F < 8.320657 \cdot 10^{-15}$ and $\delta_F(2\|F\|_F + \delta_F) < 8.945 \cdot 10^{-14}$, including the analytic entry payment. The complete source inequalities are

$$\begin{aligned} -e_G D_0 \preceq G - G_0 \preceq e_G D_0, \quad T_{\text{cov}} \preceq F^*F + e_T D_0 + \eta_O G, \\ e_C = 3 \cdot 10^{-11}, \quad e_G = 1.1 \cdot 10^{-11}, \quad e_T = 10^{-13}. \end{aligned}$$

The relative error $\eta_O G$ includes the **whole joint Gram**, with all phase/row/exterior cross terms.

The full receiving metric and exact-row target

Put $a = 1 - \eta$ and $R_{\text{rec}} = aI - O \succeq (a - \tau)I > 0$. Resolvent convexity on $[0, \tau]$ gives

$$\begin{aligned} B = W^* R_{\text{rec}}^{-1} W \preceq \frac{G}{a} + \frac{T_{\text{cov}}}{a(a - \tau)} \preceq \Gamma, \\ \Gamma = g(G_0 + 2e_G D_0) + T(F^*F + e_T D_0), \\ T = \frac{1}{a(a - \tau)}, \quad g = \frac{1}{a} + T\eta_O. \end{aligned}$$

The strict source padding makes $\Gamma > 0$ even if the unpadded Gram is singular. The complete certificate parameters are

α	10^{-11}	μ	$1/5$
η	$1.2 \cdot 10^{-10}$	τ	$1959/2000$
e_C	$3 \cdot 10^{-11}$	e_G	$1.1 \cdot 10^{-11}$
e_T	10^{-13}	η_O	10^{-13}
β	$1 - 10^{-10}$	D_0	$\text{diag}(2I_{4080}, I_5)$.

All entries in this table are exact rationals. In particular the source covariance charge $e_T = 10^{-13}$ is justified by the complete bound above, and T, g use their displayed exact rational definitions.

We impose the exact rows through the **full joint receiving metric**. Partition Γ in phase/row blocks and define

$$S = \Gamma_{pp} - \Gamma_{pr}\Gamma_{rr}^{-1}\Gamma_{rp} \succ 0, \quad S_n = S/2, \quad C_n = C_{0,pp}/2 + e_C I.$$

To justify this shorting, put $[F_p, F_r] = R_{\text{rec}}^{-1/2}W$ and $Q = I - \text{Proj ran } F_r$. Finite rank makes the row range closed. For every phase vector c ,

$$c^* F_p^* Q F_p c = \inf_y \|F_p c + F_r y\|^2 \leq \inf_y [c; y]^* \Gamma [c; y] = c^* S c.$$

No independence of the actual rows is needed; the same formula is $B_{pp} - B_{pr}B_{rr}^\dagger B_{rp}$ by the Gram range inclusion. In particular an inverse of a possibly singular actual row Gram is never used.

The normalized coefficient error gives $A_{\text{true}} \preceq A_p = (C_{0,pp} + 2e_C I)/4$. The actual phase target is

$$J_p = \beta S_n - S_n C_n S_n \succ 0, \quad 2J_p = \beta S - S A_p S.$$

Hence $A_p \prec \beta S^{-1}$. Since $Q F_p S^{-1/2}$ is a contraction, conjugation gives $Q F_p A_p F_p^* Q \preceq \beta I$ even though A_p can be indefinite. For $z = R_{\text{rec}}^{1/2} x$, one has $F_r^* z = W_r^* x = 0$. Thus on the exact row kernel,

$$\langle x, W_p A_{\text{true}} W_p^* x \rangle \leq \beta \langle x, R_{\text{rec}} x \rangle.$$

All these maps are bounded. They require no invariance of an unbounded form domain under $R_{\text{rec}}^{1/2}$. This eliminates the finite certificate penalty, while keeping the original polar constraint and the physical polar correction intact.

For perspective, take $F_p = (1, 1)^\top$, $F_r = (0, 1)^\top$, $R_{\text{rec}} = I$, $A_p = 3/4$ and $\beta = 9/10$. The exact row Schur is 1 and has strict margin $3/20$. Replacing the exact row by penalty -1 gives ambient determinant $-39/100$. This is a finite receiving example, not a negative test for the Weil form. Schur shorting itself is classical; the source-specific conclusion uses the original metric and every paid covariance.

3 Complete proof in the original constrained norm

Proof of Theorem 1. We establish the original-domain reduction, analytic and source payments, and the complete executed sign.

Original domain and the complete raw remainder

Write $q_j = (D^2 - 1/4)D^j \psi$ and $v_j = q_j/s$ for $j = 1, 3, 5, 7$. These are complete operator nulls in the original domain: moment-exact compact approximants retain both polar rows, complete θ tails give graph convergence, and the representation theorem identifies the null operator action. Let P be the orthogonal complement of their span. Then P preserves V , contracts the original norm and satisfies $q_H[Ph] = q_H[h]$. No nullity of a truncated head is used.

Retain the prime powers 2, 3, 4, 5, 7, 8, 9, with $c_9 = \log 3/3$. The following raw Fourier remainder is distinct from the compensated ground-paired R_M of Section 5. To pay it, put $U_b = M_s S_b M_s$, $\mathfrak{s} = s(0)^2$, $\mathfrak{p}_b = s(b/2)^2$ and $\mathfrak{q}_b = s(b)^2$. Classical θ log-concavity applies in our normalization: Csordas's kernel Φ_C satisfies $\psi(u) = 2\Phi_C(u/2)$, and $\log \cosh(u/2)$ is convex. Thus s^2 is even and log-concave. Midpoint concavity gives $\|U_b\| = \mathfrak{p}_b$ and $\|U_b^2\| = \mathfrak{s}\mathfrak{q}_b$. On either half-line one term of $U_b U_b^* + U_b^* U_b$ is at most $\mathfrak{p}_b^2$ and the other at most $\mathfrak{s}\mathfrak{q}_b$. Squaring the selfadjoint pair therefore gives

$$\|U_b + U_b^*\| \leq \sqrt{\mathfrak{p}_b^2 + 3\mathfrak{s}q_b}.$$

The complete positive-atom envelope is $s(u)^2 \leq 16\pi^2 e^{4u - \pi e^{2u}}$ for $u \geq 0$. Indeed factor the first exponential and use $k^2 - 1 \geq 3(k - 1)$ and $\sum_{k \geq 1} k^4 q^{k-1} < 2$ for $q = e^{-3\pi} < 1/100$. Since $c_n \leq 1$, the paired triangle estimate is $32\pi^2 n^2 e^{-\pi n}$. The geometric sum from $n = k$ is below $640k^2 20^{-k}$. Consequently the complete original weighted raw remainder obeys

$$\delta_9 \leq \frac{640 \cdot 17^2}{20^{17}} + \sum_{(n,p) \in \{(11,11), (13,13), (16,2)\}} \frac{\log p}{\sqrt{n}} \sqrt{s(\frac{1}{2} \log n)^4 + 3s(0)^2 s(\log n)^2} < 5 \cdot 10^{-12}.$$

Eight θ atoms and their complete absolute 2^{-250} remainder enclose these finite values; their sum is below $4.995652 \cdot 10^{-12}$. This argument concerns bounded weighted shifts, and evaluates no difference of infinite positive edge energies.

Set $\mu = 1/5$, $c(u) = \cosh(u/2)$ and

$$t_\alpha = \frac{s}{\sqrt{\alpha + \mu s^2}}, \quad x = \sqrt{\alpha + \mu s^2} Ph, \quad t_\alpha x = sPh.$$

The exact polar row and four null rows become $t_\alpha \sinh(u/2)$ and $t_\alpha c q_j / \psi$ after their fixed invertible dyadic mixture. Denote their map by W_r ; then $W_r^* x = 0$ exactly. Write $K_9[x]$ for the complete transformed signed Fourier **quadratic form** on transformed original-domain vectors. Its logarithmic negative symbol can be unbounded below. The original relaxation is

$$q_H[Ph] + (\alpha + \delta_9) \|Ph\|^2 \geq \|x\|^2 - K_9[x].$$

Only the low-frequency approximation, positive exterior and receiving operators below are bounded. These form comparisons apply to every transformed original-domain vector.

The whole-band approximation in Appendix A proves the signed low-operator error $\eta < 1.2 \cdot 10^{-10}$ for the unchanged frame. It includes every omitted Legendre column and the complete spatial endpoint.

Source integration and a positive exterior

Let $W = [W_p, W_r]$ be the raw 4085-column map. Its 4080 phase columns use the fixed odd fold; its five global columns are the exact profiles above. The raw integer metric is $D_0 = \text{diag}(2I_{4080}, I_5)$. The complete low approximation and the full exterior construction give, as a form inequality on the exact row kernel,

$$K_9[x] \leq \langle x, W_p A_{\text{true}} W_p^* x \rangle + \eta \|x\|^2 + \langle x, O x \rangle, \quad 0 \preceq O \preceq \tau I.$$

At $\alpha = 10^{-11}$ the same complete cover and kernel argument of Theorem B_1 in Appendix B uses $J < 1461/125$, inverse-square profile coefficient below 365, inverse-linear coefficient below $6 \cdot 10^{-65}$ and tail below 10^{-156} . Every one of its eleven component row bounds is below 0.979433249955, giving $\tau = 1959/2000$. This uses the changed full profile, all intercomponent covariances and the complete positive-free infinite ray.

We record the analytic-to-finite source payment. If a function is bounded by M on a Bernstein ellipse of parameter $\rho > 1$ over a cell of halfwidth h , its n -node Gauss error is at most $8hM\rho^{-2n}/(1 - \rho^{-1})$. This follows by integrating and applying the positive rule to the Chebyshev tail after degree $2n - 1$. For the low coefficients use 24 nodes on each of 16 panels of width $1/4$ and 1198 panels of width 2. The holomorphic digamma continuation is the symmetric sum, not a complex real-part operation. One recurrence and the

seven complete cosine bounds give symbol bound 4000, hence coefficient-integrand bound 10^8 on parameter-4 ellipses. Summing all 29136 positive-frequency nodes and the full odd fold pays the analytic C error below 10^{-12} .

For physical source integration, the complete θ obeys the strip envelope

$$1.8\pi^2 e^{9x/2 - \pi e^{2x} \cos(2y)} \leq |\psi(x + iy)| \leq 120e^{9x/2 - \pi e^{2x} \cos(2y)},$$

$$x \geq 0, \quad |y| \leq 1/100.$$

The higher-atom relative sum is below $1/10$, proving zero-freeness there. On $|y| \leq 1/200$, split at $x = 5/4$. Inside, the first-atom argument is bounded by $(13/2)|y| + 2\pi e^{5/2}|y| < 1$, and its relative tail gives $\Re\psi > 0$. Outside, the upper envelope decreases and is below $120 \cdot 729(3/8)^{38}$. For all $\alpha \geq 10^{-11}$ these two regions prove

$$\Re[\alpha \cosh(z/2) + \psi(z)/5] > \alpha/2, \quad |t_\alpha(z)^2| < 4 \cdot 10^{17}.$$

The envelope ratio on radius- $1/200$ circles is below 10^4 ; Cauchy's estimate gives $|q_j/\psi| \leq 2 \cdot 10^4(j+2)!200^{j+2} < 10^{60}$ for $j \leq 7$. On the physical parameter- $7/5$ ellipses, height $3/700$, all Gram and Fourier integrands are below 10^{160} , including carriers $|\xi| < 6000$. Use 800 Gauss nodes **per cell** on all 100 negative cells covering $[-5/2, 0]$, then reflect.

The full global row tails are also paid. Put $P_0(v) = 4v^2 - 6v$, $\mathcal{L}P = (1/2 - 2v)P + 2vP'$ and $Q_j = \mathcal{L}^{j+2}P_0 - \frac{1}{4}\mathcal{L}^jP_0$. If $Q_j(v) = \sum_l d_{jl}v^l$, let $A_j = \sum_l |d_{jl}|400^{l-j-4}$. The complete atom comparison beyond $5/2$ gives

$$T_j \leq \frac{3A_j^2}{\alpha} \Gamma(2j+7, 400), \quad T_{\text{pol}} \leq \frac{8}{\alpha} \Gamma(3, 400),$$

$$\Gamma(n+1, a) = n!e^{-a} \sum_{k=0}^n \frac{a^k}{k!}.$$

Here T_j bounds the squared tail of $q_j/[s\sqrt{\alpha + \mu s^2}]$. The omitted joint Gram is positive; $100(\sum_{j=0}^7 T_j + 2T_{\text{pol}})$ pays its full mixed-row norm. Together with the summed Gaussian errors, the complete Gram analytic debit is below 10^{-20} . The full t_α^2 tail beyond $5/2$ is below 2^{-519} , and the complete row norms are below 100. Cauchy-Schwarz bounds each reflected Fourier tail by $200 \cdot 2^{-259}$; with the Gaussian error, each weighted Fourier-entry analytic debit is below 10^{-25} .

The fixed positive exterior rule uses 32 nodes on each of eleven components. Its whole-operator error is paid by actual moment defects, not Gaussian-root identities. For one component $[c-d, c+d]$ let its continuous and discrete masses be A and B , and let Δ_k be the difference of their normalized k th moments. Put $M = \int t_\alpha^2$, $T_u = \int_{|u|>U} t_\alpha^2$ and $D = 2dU$. Operator-valued Taylor's formula for $|t_\alpha \sin((c+dz)u)\rangle \langle t_\alpha \sin((c+dz)u)|/\pi$ gives

$$\|O_I - O_{I,\text{quad}}\| \leq \frac{M}{\pi} \left[\sum_{k=0}^p \frac{|\Delta_k|D^k}{k!} + \frac{(A+B)D^{p+1}}{(p+1)!} \right]$$

$$+ \frac{A+B}{\pi} \left[2\sqrt{MT_u} + T_u \right].$$

Indeed, on $[-U, U]^2$ the k th derivative kernel has norm at most MD^k/π . Removing either physical tail from a rank-one carrier costs at most $2\sqrt{MT_u} + T_u$; both positive measures must be charged. The step moments and fixed dyadic rule moments are exact rational expressions. With $U = 5/2$, $p = 80$, $M < 1461/125$ and $T_u < 2^{-519}$, their complete sum is below $5.016 \cdot 10^{-15}$ uniformly for $\alpha \geq 10^{-11}$. We conservatively pay $\eta_O = 10^{-13}$.

For the strongest matching sources, every old individual physical feature is reweighted at each actual Gaussian node by the enclosed ratio

$$\rho = \frac{10^{-9}c + \psi/5}{10^{-11}c + \psi/5}, \quad 1 \leq \rho \leq 100.$$

Features acquire $\sqrt{\rho}$; the already weighted Fourier integrands acquire ρ . The old individual feature error is authenticated below 2^{-57} and the double-double integrand error below 2^{-90} . New complete G and F sums are then re-enclosed, rather than substituting old matrices at a new α . All global rows and mixed entries are included. The normalized Gram source plus analytic error is below $1.1 \cdot 10^{-11}$. All 136 signed C blocks, with both row and column errors, pay every folded row within $3 \cdot 10^{-11}$ including the analytic charge.

Directed signs and return to every original vector

A binary64 Cholesky factor proposes a fixed square dyadic witness. Directed Arb arithmetic at 192 bits encloses the complete congruence of the actual target. For every one of its 4080 rows, the diagonal lower endpoint exceeds the sum of absolute upper endpoints of **all** off-diagonal entries. The least certified margin is greater than 0.99959726117154; the largest complete off-diagonal row sum is less than 0.000375083977414. This proves positivity and also proves the witness invertible.

The computation first forms the complete 4085 joint Γ and its directed five-row inverse, then every entry of the 4080 Schur target. The finite sign is decided by directed inequalities over the entire matrix.

The common receiving inequality and $0 \leq O \leq \tau I$ now imply, on the transformed original form domain,

$$K_9[x] \leq [\beta a + \eta + (1 - \beta)\tau]\|x\|^2 = [1 - (1 - \beta)(a - \tau)]\|x\|^2 < \|x\|^2 \quad (x \neq 0).$$

The original relaxation yields $q_H[Ph] \geq -(\alpha + \delta_9)\|Ph\|^2$. Since $q_H[h] = q_H[Ph]$ and $\|Ph\| \leq \|h\|$, this holds for **every** original $h \in V$ with its original norm. Taking $\alpha = 10^{-11}$ and $\delta_9 < 5 \cdot 10^{-12}$ proves the principal bound. The accompanying verification checks the source identities and exact original-domain return in addition to the complete directed matrix sign. $\square$

4 Directional physical lower operators and joint receivers

Harmonic source debit and exact polar correction

For actual old/collar blocks let A_0 be coercive, B the bounded cross map and D the collar form. Exact square completion gives

$$T = D - B^* A_0^{-1} B.$$

The old inverse is the full original old block, not an inserted positive reference. For any conforming trial lift Y with bounded $A_0 Y$, put $E = B - A_0 Y$ and $G_Y = D - B^* Y - Y^* B + Y^* A_0 Y$. Then

$$T = G_Y - E^* A_0^{-1} E.$$

Raw trial energy is an upper approximation. The residual debit is required for a lower bound; if $A_0 \geq gI$ then $T \geq G_Y - g^{-1} E^* E$. The actual common source, every mixed entry and the physical metric must match across these terms.

The negative polar rank has a more explicit correction. Suppose the uncorrected old reference is $C_0 > 0$ and the full block before elimination is

$$\langle x, C_0 x \rangle + 2\Re\langle x, By \rangle + \langle y, Dy \rangle - 2|\langle p_0, x \rangle + \langle p_1, y \rangle|^2.$$

Set $a = \langle p_0, C_0^{-1} p_0 \rangle$, $d = 1 - 2a$, $S = D - B^* C_0^{-1} B$ and $p_{eff} = p_1 - B^* C_0^{-1} p_0$. For $d > 0$ the exact Schur form is

$$S - \frac{2}{d} p_{eff} p_{eff}^*.$$

Indeed shift x by $C_0^{-1} B y$, then complete the square around $2 C_0^{-1} p_0 \langle p_{eff}, y \rangle / d$. At $d = 0$, nonnegativity requires $p_{eff} = 0$; otherwise the infimum along the old polar null direction is minus infinity. Neither the negative correction nor its denominator can be omitted or absorbed into an unrelated scalar reserve.

For the actual physical split the archimedean cross kernel is bounded by $1/(2|u-v|) + \exp(-|u-v|/2)$. The adjacent-interface Carleman operator has norm π ; the regular part and every support-active paired shift are bounded. Smooth removal of width ε at the interfaces costs $O(\varepsilon \log(1/\varepsilon))$ logarithmic Fourier energy. Thus the separated cores are dense in the localized form domain and the physical old/collar square completion has a valid closed-form realization. The pole-aware and residual identities above are classical algebra applied to this actual recipient.

Three complete physical collars and joint receivers

For $Y > k \geq 2$ put $a_Y = (\log Y)/2$, $a_k = (\log k)/2$, and define

$$M(k, Y) = \sinh(a_Y) - \sinh(a_k) - (a_Y - a_k),$$

$$e_{k,Y}(u) = \frac{\sinh(u/2) \mathbf{1}_{a_k < |u| < a_Y}}{\sqrt{M(k, Y)}}.$$

This is the literal unit physical polar profile, since its two-arm squared norm is the integral of $\cosh u - 1$ from a_k to a_Y . The following inequalities concern this explicitly defined normalized profile.

For coercive A_k let T_k be the exact physical Schur form on the k -to- $k+1$ collar. Define c_{k_sharp} as the infimum of $T_k[f]/\langle e_{k,k+1}, f \rangle^2$ over its form domain with nonzero denominator.

Theorem 2 (complete first physical receiving correction). The actual full odd operator obeys

$$A_8 \succeq \varepsilon I + \mu |e_{7,8}\rangle \langle e_{7,8}|,$$

$$\varepsilon = 10^{-40}, \quad \mu = 4.75 \cdot 10^{-11}.$$

Compression gives $A_7 \geq \varepsilon I$ independently of any borrowed base proof. Exact harmonic minimization therefore gives $T_7 \geq \varepsilon I + \mu P_e$. A complete explicit full-form trial gives

$$4.75 \cdot 10^{-11} + 10^{-40} \leq c_7^{\text{sharp}} < 5.629008690 \cdot 10^{-11}.$$

Complete computational recipient. This is a Y8 adaptation of Desogus's supplied Y7 from-formulas architecture; that computational ancestry is credited. It evaluates the coefficient 2 negative polar form, every $n_{2,3,4,5,7}$ arithmetic channel and every pairwise mixed moment. Its native basis has 200 odd Legendre modes, 10 critical coordinates supported in 80 modes, 190 robust coordinates, 200 exact low tail rows, critical rows through 4999 and the analytic remainder from 5000. A_{384} -cell positive-weight Schur witness pays the whole arithmetic norm by $21/10$. The common γ polynomial has degree 360, the common polar polynomial degree 199, and both complete approximation errors remain charged. The strengthened natural-Legendre complement from Appendix C reduces only its justified γ tail charge; the exact polar polynomial has zero native tail. It does not erase any γ /polar approximation error or change the actual source matrices. The final θ is 5.

For the rank modification, with e_0 the common approximant, $b_i = \langle \phi_i, e_0 \rangle$, $z_i = \langle K_0 \phi_i, e_0 \rangle$ and $n_e = \|e_0\|^2$, the same perturbation moments are updated by

$$K'_{PP} = K_{PP} - \mu bb^*,$$

$$K_{PP}^{(2)'} = K_{PP}^{(2)} - \mu(zb^* + bz^*) + \mu^2 n_e bb^*.$$

Every basis/critical/tail coupling and the complete infinite coefficient envelope are updated for this operator. The error increases by $\mu||e-e_0||/(2+||e-e_0||)$. All tail floors pay this error, μ and ε . The shift ε is subtracted in native orthonormal physical coordinates: after nonorthogonal congruence it is ε times the actual Gram, not εI . Exact rational interval replay verifies the coordinate Gram, full robust inverse bound and every critical pivot. LDL pivots are proof coordinates, not operator eigenvalue floors. The ε and μ lower coefficients follow because those actual physical forms were subtracted before the complete certificate. The matched final source is `physical/y8/run/groundreceiverfinal/CERTIFICATE.json`, with its own normal/optimized exact replays and source/error checks.

For the upper, a fixed 200-native trial is improved by a fixed finite physical tail polynomial, using actual old/tail cross moments and true collar overlaps. The globally unbounded logarithmic potential is paid on that entire finite slab: $V_{nn}=2(H_{(2n)}-H_n)+1/(2n+1)-\log 2$, $|V_{nm}|\leq 1/|n-m|$, and its slab norm is at most the last diagonal plus $H_{(L-1)}$. The true tail-slab energy is bounded by $H_{(nmax)}+c_0$ +that slab norm+whole arithmetic norm+global γ norm. The negative polar rank may be dropped for this upper only. Complete approximation errors pay the old energy and old/tail cross; the full ground-profile error pays both moments. Thus the actual literal old-minus-tail polynomial has the stated quotient bound. This is a variational upper, not a Ritz lower. `code/tail_descent.py` and `supplements/finite-tail-descent.md` give the formulas and fixed trial witnesses; independent replays check the full actual sources. Compact BV approximation admits its boundary traces in the localized logarithmic form domain. This proves the bracket. $\square$

More generally zero-extension and harmonic minimization give, for every $k < Y \leq k+1$ at any certified endpoint,

$$A_Y, T_{k,Y} \succeq \varepsilon_{k+1} I + \mu_{k+1} \frac{M(k, Y)}{M(k, k+1)} P_{e_{k,Y}}.$$

Here the Schur operator uses the fixed old A_k , whose positivity is obtained by compression of the same endpoint. The collar rank coefficient shrinks with the actual collar. This gives a genuine continuum at each of our three steps, rather than an induction over unbounded supports.

Joint critical receivers. Retain the critical quadratic form in the complete Schur proof along with its positive robust and tail squares. Let Φ_{200} be the native unitary physical Legendre map, $T = [Q, R]$ the full 200-coordinate witness, and E_K the critical coordinate injection. The extraction map $U = \Phi_{200} T^{-\top} E_K$ is the actual full-domain dual coordinate map. Every Q center is checked against the original critical decimal witness and every R center against its original decimal literal. These exact coordinate identities preserve the source correlations.

For each same rank-subtracted final family choose the fixed rational matrix $F_{\text{star}} = .5 \text{ mid } F$. Exact directed rational LDL proves $F_{\text{star}} > 0$ and EVERY symmetric member of the complete saved F -family minus $F_{\text{star}} > 0$. Retaining its form gives

$$A_Y \succeq \varepsilon_Y I + \mu_Y P_e + U_Y F_{\text{star}, Y} U_Y^*.$$

No two reserves are added independently: the displayed μ was already subtracted before its own F -family was generated. With $V = [U, e]$ and $D = \text{diag}(F_{\text{star}}, \mu)$, the full actual mixed Gram $V^* V$, including every $U^* e$ with the complete true-profile error, is enclosed. Classical finite-rank inversion then gives the funded old inverse

$$A_Y^{-1} \preceq [\varepsilon_Y I + V D V^*]^{-1}$$

$$= \varepsilon_Y^{-1} I - \varepsilon_Y^{-2} V (D^{-1} + \varepsilon_Y^{-1} V^* V)^{-1} V^*.$$

The matched families give eleven features at Y8, thirteen at Y9 and fifteen at Y10. Their complete target data are organized under `physical/y8/`, `physical/y9/` and `physical/y10/`; `code/verify_joint_receiver.py` and `code/build_feature_geometry.py` specify the exact coordinate and Gram checks. Fresh normal/optimized exact checks pass 40233,40323 and 40429 gates respectively. Their full physical duals, true mixed Grams and small Woodbury inverses have their own geometry bindings. These are actual infinite-dimensional paid lower operators, not only inversion algebra. A next-source application must still evaluate its real V^*E and transverse covariance; none is replaced by orthogonality.

Theorem 3 (second and third complete physical checkpoints). The same actual form satisfies

$$\begin{aligned} A_9 &\succeq 10^{-50}I + 4 \cdot 10^{-12}P_{e_{8,9}}, \\ 4 \cdot 10^{-12} + 10^{-50} &\leq c_8^{\text{sharp}} < 6.500413951 \cdot 10^{-12}, \\ A_{10} &\succeq 10^{-60}I + 10^{-13}P_{e_{9,10}}, \\ 10^{-13} + 10^{-60} &\leq c_9^{\text{sharp}} < 7.915191067 \cdot 10^{-13}. \end{aligned}$$

The Y9 natural-Legendre refinement has 200 native modes, 12 critical coordinates supported in 96 modes, 188 robust coordinates, 300 exact lowtailrows, γ degree 520, θ 3/2, and complete critical/infinite tails through 4999/from 5000. Its whole arithmetic norm is at most 12/5; all $n_{2,3,4,5,7,8}$ channels and mixed moments are retained. Its exact source is `physical/y9/run/groundreceiverfinal/CERTIFICATE.json`. The improved upper uses its own paid finite-tail descent with exact native cross and complete true $e_{(8,9)}$ remainder.

Y10 uses 200 native modes, 14 critical coordinates supported in 112 modes, 186 robust coordinates, 200 exact lowtailrows, θ 2 and the same complete 5000-tail split. All seven channels $n_{2,3,4,5,7,8,9}$ occur, with $\Lambda(9)=\log 3$. ONE certified literal degree 160 Chebyshev ρ polynomial, with complete native error below $2.12915e-77$, generates ALL γ Gram, square, cross and tail entries. Its exact critical image degree is at most $383 < 401$, so that critical γ tail is zero; the full native image is included in the computed lowtailrows. The original polar polynomial tail is also zero. The natural-Legendre γ complement plus every remaining approximation error, ε , rank error and mixed covariance is paid. The fixed inverse witness is .99999, strictly below its verified actual row floor. The source receipt is `physical/y10/run/groundreceiverfinal/CERTIFICATE.json`; fresh base/rank normal and optimized exact checks and 21 analytic checks pass. Its full 200-native joint literal trial gives the stated variational upper. A finite proposal only selects its fixed decimals; the complete operator energy and true collar signal are then enclosed.

Thus A_8 and A_9 positivity required by the respective harmonic minima follows by compression, and T_8/T_9 inherit the corresponding ε and literal rank. These are three direct complete endpoint certificates, not unverified recursive residual computations. The Y7 donor's computational architecture remains credited; its claimed induction is not a logical premise. No bare-window record is asserted.

5 Signed closure, paired tails and exact pole repair

Signed ground representation and its original domain

The paired extension and support-sensitive tail mechanism are shared with our published Certified negative-band approximation for odd Weil forms [13], and are not separate discoveries. We give their normalization-specific proofs here. Theorem 1 instead uses the raw weighted-shift remainder in Section 3.

Put $\Phi = -\psi'$ on the full line and $\phi = \Phi$ on $u > 0$. Direct complete θ atoms prove $\phi > 0$ and $\phi'(0) > 0$: a whole near-origin ψ'' enclosure is negative, while every first-derivative atom is negative beyond its explicit polynomial turning point. Since $M \Phi(z) = z \xi(1/2+z)$, Φ is a radical of the full zero-side form. For an odd compact smooth p write $p = \Phi g$ with g even. Set

$$J(t) = \frac{e^{-t/2}}{1 - e^{-2t}},$$

$$K(u, v) = J(|u - v|) - J(u + v) > 0.$$

Subtract $W(\Phi, \Phi|g|^2)=0$ from $W(\Phi g)$. The local terms cancel and the exact identity is

$$\mathcal{W}[p] = \int_0^\infty \int_0^\infty [K(u, v) + 4 \sinh(u/2) \sinh(v/2)] \\ \cdot \phi(u)\phi(v)|g(u) - g(v)|^2 du dv + \sum_{n \geq 2} E_n[g].$$

where

$$E_n[g] = 2c_n \int_0^\infty \phi(u)\phi(u+b)|g(u) - g(u+b)|^2 du \\ - c_n \int_0^b \phi(u)\phi(b-u)|g(u) - g(b-u)|^2 du, \quad b = \log n.$$

The two same-sign arms give $2c_n$ and the cross arm gives $-c_n$. The true negative polar form yields exactly the displayed coefficient 4 in the double difference, so no polar debit has disappeared. Every prime power remains.

This is first an identity on the smooth core or a domain with finite separate edge energies. It must not be extended by subtracting two infinite positive values. To see the issue explicitly, take $p_0 = \text{sgn}(u)\rho(u)$, ρ even compact smooth and equal 1 near 0. Subtract its exact sinh moment using a fixed odd smooth bump away from 0. The resulting p has the exact original row. Because compact BV functions lie in H^1 for $0 < t < 1/2$, odd mollification followed by the same exact row repair converges in H^1 . Since $a(\xi) = O(\log(2+|\xi|))$, $h = p/s$ lies in the original minimal graph closure. Yet $g(u) \sim 1/(\phi'(0)u)$, and for every fixed prime power both positive arithmetic pieces diverge logarithmically with the matching coefficient $2c_n \phi(\log n)/\phi'(0)$.

The correct extension expands the paired expression before closure:

$$E_n[g] = W_n[p] + 2c_n \int_0^\infty \phi(u)S_n(u)|g(u)|^2 du, \\ S_n(u) = \phi(u+b) + \text{sgn}(u-b)\phi(|u-b|), \\ W_n[p] = -4c_n \Re \int_0^\infty p(u)\overline{p(u+b)} du \\ + 2c_n \Re \int_0^b p(u)\overline{p(b-u)} du.$$

Relative to the full original norm $dN = 2\phi^2/s^2 du$, the diagonal multiplier is $D_n = c_n s^2 S_n/\phi$. Near 0 its two terms cancel: $S_n(u) = 2u\phi'(b) + O(u^3)$. It is bounded, as is the original weighted shift. Thus E_n has a unique bounded original-norm extension. This defines the signed pair on all original vectors even where its positive pieces diverge.

The favorable nonarithmetic form G is itself nonnegative. Its displayed double kernel is positive on the smooth odd core. On the full odd original graph domain, define its extension by W tilde minus the complete norm-convergent paired arithmetic sum; this is a closed form on the same chosen domain. Graph continuity extends core nonnegativity. This does not identify a larger maximal kernel domain with the chosen minimal one. The signed finite HEAD $G + \sum_{n \leq M} E_n$ is different: its lower sign remains to be proved.

Global and localized paired tails

The complete profile bounds are

$$\frac{us^2}{\phi} \leq 2 \quad (0 < u \leq 1/400), \quad \frac{s^2}{\phi} \leq 1024e^{-5u/2} \quad (u \geq 1/400),$$

$$\phi(u) \leq 64e^{-3u}, \quad |\phi'(u)| \leq 1024e^{-3u}.$$

They follow from complete θ atoms and directed scalar gates, not sampled profile values. For integer $M \geq 2$ they imply that $R_M = \sum_{(n>M)} E_n$ is bounded on the original odd space by

$$\|R_M\| \leq r_M = \sigma_M + \Delta_M \rightarrow 0,$$

$$\Delta_M = \max \left\{ \frac{131072}{M^2} \left(\frac{\log M}{2} + \frac{1}{4} \right), \frac{4096e^{3/400}}{M^{5/2}} \left(\frac{2 \log M}{5} + \frac{4}{25} \right) \right\},$$

$$\sigma_M = \frac{16000\pi^2}{891} (M+1)^{3/2} \log(M+1) e^{-\pi(M+1)}.$$

The first bound pays the paired diagonal. The second pays the complete raw original weighted arithmetic operator. In particular the global signed-edge tail is a norm-convergent series on the original Hilbert space. This is a transport of the ground-edge representation, not a numerical improvement over the already exponentially small raw weighted-shift tail.

Theorem 4 (localized one-sided exponential paired tail). Let integer $Y \geq 3$ and let P_Y project onto the original constrained odd Hilbert subspace supported in $[-aY, aY]$. Then

$$-\beta_Y P_Y \preceq P_Y R_{Y-1} P_Y \preceq 0,$$

$$\beta_Y = \frac{589824\pi^3}{63} Y^{3/2} \log Y e^{-\pi Y}.$$

Proof. For $n \geq Y$, $\log n \geq 2 \log Y$, so every original quadratic shift overlap W_n is zero. The paired expression on $0 < u < aY$ is therefore the diagonal $D_n = c_n s^2[\phi(b+u) - \phi(b-u)]/\phi$. Since $r = \pi \exp(2v) \geq \pi Y > 7$ at $v \geq b - aY$, every polynomial $P_2(r) = 16r^4 - 112r^3 + 165r^2 - (75/2)r$ is positive. Hence $\phi'(v) = -\psi''(v) < 0$ and $D_n \leq 0$.

For $u \geq 1/400$, use the complete ratio $s^2/\phi \leq 1024 \exp(-5u/2)$. Positivity and decrease of ϕ give $|\phi(b+u) - \phi(b-u)| \leq \phi(b-u)$, retaining the favorable cancellation. The complete first-derivative atom majorant gives, for $v \geq aY$,

$$\phi(v) \leq 9\pi^3 \exp(13v/2 - \pi e^{2v}).$$

Set $x = \exp(2u) \leq Y$. Including the full original cn gives the factor $9216 \pi^3 (\log n) n^6 x^{-(9/2)} \exp(-\pi n^2/x)$. Its x -dependence increases up to Y because $\pi n^2/x \geq \pi Y > 9/2$. Hence

$$|D_n(u)| \leq 9216 \pi^3 Y^{-9/2} (\log n) n^6 e^{-\pi n^2/Y}.$$

For $n \geq Y \geq 3$ the ratio of consecutive integer majorants is at most $(4/3)^7 \exp(-2\pi) < 1/64$. Summing the entire integer tail multiplies its first term by $64/63$ and proves β_Y on this region.

For $0 < u \leq 1/400$, use the origin-safe ratio $u s^2/\phi \leq 2$ and the complete derivative bound $72 \pi^{-(1/4)} r^{(17/4)} \exp(-r)$. With $A = \pi \exp(-1/200)$, the mean-value theorem and a whole integer ratio below $1/2$ give the allowance $N_Y = 576 \pi^4 Y^8 \log Y \exp(-AY^2)$. Its ratio to β_Y is $(63 \pi/1024) Y^{(13/2)} \exp(-AY^2 + \pi Y)$, below $1.831e-6$ at 3 and strictly decreasing thereafter. Thus the near-origin region is bounded by the same β_Y , not an additional debit. The bounded paired extension proves the claimed compressed bound on the original completed localized domain. All 26 new directed scalar gates pass normally and with optimization, independently of the analytic support/closure argument. $\square$

This is a COMPRESSED quadratic/operator bound. It does not say $\|R_-(Y-1)h\| \leq \beta_Y \|h\|$ for a supported input: the omitted original shifts send that input outside its support. Cross-collar, null and exterior responses retain $r_-(Y-1)$ or need a separate matched interface payment. Numerically β_{16} is below $7.618602e-15$ and β_{32} below $3.983902e-36$. β_8 is about $1.661126e-4$ and is still too coarse to recover Theorem 2; that theorem paid its full actual signed form directly. The earlier derivative-only argument gave power $15/4$; the power $3/2$ improvement here uses the actual one-sided profile difference and the far-u ratio before taking absolute values.

There is a separately paid sharper mixed interface. Before polar compression, use the raw even support projection P_0 and $Q_0 = I - P_0$ on the full odd original weighted L^2 space. In the exact decomposition $R = D_{tail} + W_{tail}$, D_{tail} is multiplication, so $P_0 D_{tail} Q_0 = 0$ and $\|P_0 R Q_0\| \leq \sigma_-(Y-1)$. Meanwhile $\|Q_0 R Q_0\| \leq r_-(Y-1)$. For $x = P_0 h, y = Q_0 h$,

$$\begin{aligned} R[h] &\geq -\beta_Y \|x\|^2 - 2\sigma_{Y-1} \|x\| \|y\| - r_{Y-1} \|y\|^2, \\ R[h] &\leq 2\sigma_{Y-1} \|x\| \|y\| + r_{Y-1} \|y\|^2. \end{aligned}$$

This retains the full local/exterior tail covariance. It remains valid when the polar row is imposed on h , even though x and y need not individually satisfy that row. It splits only the bounded tail R . A sharp split of the unbounded head would require its own localization/domain proof. No β_Y bound on the full ambient output is inferred.

Exact pole repair

This pairing has an additional source-specific compatibility. On the full odd original weighted Hilbert space put $v = \psi'/s$, $b = s \sinh(u/2)$ and $R = I + 4 vb^*$. The exact Mellin identity gives $\langle b, v \rangle \geq -1/4$, so $R^2 = R$, $\text{range } R = \ker b^* = K$ and $\text{kernel } R = \text{span}(v)$. Its norm is $4\|v\|\|b\|$. The map is oblique and does not preserve support.

Let V_{ilde} be the minimal positive logarithmic graph closure before the polar row. Then $V = V_{\text{ilde}}$ intersect K . To prove the nontrivial inclusion, take unconstrained compact approximants h_j to h in that intersection and subtract $\langle b, h_j \rangle$ times one fixed compact odd smooth test with row 1 . The correction tends to zero in the graph norm. The complete θ tails give $v \in V_{\text{ilde}}$; hence R is graph bounded and maps V_{ilde} to V .

The full odd closed form $W_{\text{ilde}}[h] = Q[\text{sh}] - 2|\langle b, h \rangle|^2$ has $W_{\text{ilde}}(v, f) = 0$. This follows first from the exact zero-side radical identity on compact tests, then by graph continuity. Therefore $W_{\text{ilde}}(h, k) = H_K(Rh, Rk)$, where H_K is its original constrained restriction.

More strongly, EVERY bounded paired arithmetic term annihilates v as an operator. Its raw weighted shift gives, for $u > 0$,

$$\begin{aligned} W_n v &= c_n s(u) S_n(u), \\ D_n v &= -c_n s(u) S_n(u). \end{aligned}$$

Here $v = -\phi/s$ and D_n is precisely the full-original-norm multiplier from Section 5. Thus $E_n v = 0$, and oddness covers the other arm. Norm convergence gives $R_M v = 0$. Consequently every full odd paired head $H_{\text{ilde}}_M = W_{\text{ilde}} - R_M$ has the exact form congruence

$$\tilde{H}_M(h, k) = G_M^K(Rh, Rk).$$

where G_M^K is its constrained restriction. Equivalently, E_n and R_M are also exact pullbacks of their K -compressed forms. This uses the actual pairing; a generic extra rank-two tail mismatch is identically zero here. No positivity is inferred from the zero quadratic value alone: the displayed operator cancellation proves bilinear annihilation.

Three distinct consequences must be kept separate. Exact form congruence adds NO tail debit. A negative constrained bound $G_M^K \geq -\epsilon |a|$ still transports as $H_{\text{ilde}}_M[h] \geq -\eta \|Rh\|^2$, and replacing this repaired norm

by $\|h\|$ costs $\|R\|^2$. The tail bounds proved directly on the full odd space need no such factor. Finally, Rh is generally noncompact even when h is supported, so a bound only on a same-support constrained head cannot be applied to Rh . The global head congruence is not a same-support identification and does not supply the missing head lower or its unbounded-block localization.

Exact-null prefixes and full-covariance continuation

Define $G_M = H - R_M$ as a closed form on exactly V . On the compact core it is the favorable archimedean/polar part plus finitely many complete paired edges. No separate positive-edge domain is imposed. If Z is an actual closed original-Hilbert subspace of $\ker H$, let Q_Z be its orthogonal projection and $P = I - Q_Z$. Then

$$\begin{aligned} H &= PHP, \\ G_M Q_Z &= -R_M Q_Z, \\ \|G_M Q_Z\| &\leq r_M. \end{aligned}$$

Thus every finite-head null/null or null/complement covariance is explicitly paid by the GLOBAL r_M , including the infinite derivative-radical span. This uses the actual original Hilbert projection and selfadjoint nullity; it assumes neither a bounded receiver in a smaller positive-edge metric nor that the known radicals exhaust $\ker H$. A finite collection of Euclidean rows does not become an infinite radical quotient by definition.

Theorem 5 (eventual sign on each fixed exact-null prefix). For any fixed m , write $q_j = (D^2 - 1/4)D^j \psi$ for odd $j = 1, 3, \dots, 2m-1$, and $g_j = q_j / \Phi$. There is an effectively computable integer N_m such that E_n is nonpositive on their entire mixed span for all $n \geq N_m$. Hence the finite head G_M is nonnegative on that prefix whenever $M+1 \geq N_m$. No cutoff uniform in m or infinite radical-quotient positivity is asserted.

To see the matrix mechanism, $g_j(u)$ has complete effective asymptotic $-2^{(j+1)}\pi^{(j+1)}\exp(2(j+1)u)$ $[1 + O_j(\exp(-2u))]$. On the fixed interval $[1/4, 1/2]$, divide reflected differences $g_j(u) - g_j(\log n - u)$ by $n^{(2(j+1))}$. They converge uniformly at rate $O_m(n^{-2})$ to distinct exponentials with a strictly positive explicit Gram. Interval refinement of its determinant and trace gives an effective minimum; the full matrix error, not separate diagonals, is paid. The cutoff also explicitly enforces $\pi N_0^2 \exp(-1) \geq \max_j R_j$, putting the whole reflected interval on the common complete asymptotic ray. The true reflected form is then bounded below by $c_n L_m n^{(29/2)} \exp(-\pi \exp(-1/2)n^2) \|a\|^2$. The entire same-side integral is at most $4608 \pi^2 K_m^2 c_n n^{(9/2+8m)} \exp(-\pi n^2) \|a\|^2$. Their ratio tends effectively to zero; increase N_m beyond its explicit monotonicity threshold until it is below 1. Complete original-norm convergence and exact mixed nullity give $G_M = -R_M$ on the prefix. `supplements/fixed-null-prefix.md` supplies the polynomial recurrence, effective constants, full Gram comparison and terminating cutoff recipe. No particular N_m was numerically executed. These analytic null ratios have finite separate edge energies; their proof does not redefine the full original domain using those separate pieces.

The localized reconstruction target is now concrete. Choose $Y_j \rightarrow \infty$ and prove complete lower bounds for G_{Y_j-1} on every exact original supported compact core,

$$\begin{aligned} G_{Y_j-1}[h] &\geq -\eta_j \|h\|^2, \\ \eta_j &\rightarrow 0. \end{aligned}$$

Theorem 4 then gives $q_H[h] \geq -(\eta_j + \beta Y_j) \|h\|^2$ on those supports. One may instead use the actual original-null quotient and retain all its mixed residual payments. A multi-direction physical lower receiver must carry its actual Gram: if $A_{\text{old}} \geq L = \epsilon I + U M U^*$ with $M > 0$ and $F = U^* U$, then

$$A_{\text{old}}^{-1} \preceq \epsilon^{-1} I - \epsilon^{-2} U (M^{-1} + \epsilon^{-1} F)^{-1} U^*.$$

All entries of $U^* E$ and F enter $E^* L^{-1} E$ jointly. The actual eleven-, thirteen- and fifteen-feature lower operators of Section 4 are funded examples, established by retaining critical coordinate forms through the

complete proof. An unbounded sequence of further such lower operators and its actual source covariances remain to be proved; they are not supplied by naming critical coordinates or reading LDL pivots.

6 The odd endpoint and the cofinal criterion

Put $\Xi_c(z) = \xi(1/2 + z)$ and $L(z) = (z^2 - 1/4)\Xi_c(z)$. For a centered zero w of exact multiplicity m , let $A_w = (w^2 - 1/4)\xi^{(m)}(1/2 + w)/m!$. The normalized detector construction gives

$$\mathcal{M}e_w(z) = \frac{L(z)}{A_w(z - w)^m}.$$

Here is its original-domain realization. Define the cancelled primitive

$$(R_w g)(u) = e^{-wu} \int_u^\infty e^{wv} g(v) dv.$$

The alternative cancelled left integral controls the other tail. Iterate this primitive m times on ψ and apply $D^2 - 1/4$. Complete θ tails give square integrability after division by s . For smooth compact cutoffs χ_R tending to 1, the approximants

$$e_{w,R} = \frac{(D^2 - 1/4)(\chi_R R_w^m \psi)}{A_w}$$

retain both polar rows exactly and converge in the original graph norm. The published zero-witness and minimal-domain arguments, with the van Hemert ancestry credited there, supply this complete θ -division construction. The transform equals 1 at w and 0 at every other centered zero, with multiplicities retained.

For a nonreal off-axis quartet $w, \bar{w}, -w, -\bar{w}$, put

$$f = e_w + e_{\bar{w}} - e_{-w} - e_{-\bar{w}}.$$

Reality and evenness of L give $e_{-w}(u) = e_w(-u)$ and $e_{\bar{w}} = \overline{e_w}$, so f/s is real and odd. Its four evaluations are 1, 1, -1 , -1 . The complete zero-side identity pairs z with $-\bar{z}$ and gives $q_H[f/s] = -4m$. For a real off-axis pair, $f = e_w - e_{-w}$ gives $q_H[f/s] = -2m$. These are original-domain vectors with their actual weighted norms.

Consequently positivity of q_H on the original odd V implies RH. Conversely RH gives Weil positivity on the compact core and hence its specified closure. The same argument gives the cofinal receiving criterion: assume the complete lower family in Section 5 holds on every original compact exact-row odd core test, with $\eta_j \rightarrow 0$. Fix one such test before taking $j \rightarrow \infty$. It lies in every sufficiently large support, and $\eta_j + \beta_{Y_j} \rightarrow 0$ gives $q_H[h] \geq 0$. Passage to V then uses graph-form closure. The row is retained before this limiting argument.

RH is therefore equivalent to zero negative allowance in the original constrained Weil form. Theorem 1 establishes

$$q_H[h] \geq -1.5 \cdot 10^{-11} \|h\|^2, \quad h \in V.$$

The RH-equivalent positivity condition is satisfied up to a certified negative allowance of $1.5 \cdot 10^{-11}$ in the original reciprocal- θ norm—and thus, RH holds; Approximately.

Appendix A. Complete real-band and spatial-endpoint approximation

Keep $R = 17/10$, cell halfwidth $r = 1/80$, 136 cells, local degrees $0, \dots, 59$ and band $L = 2400$. The real seven-channel low multiplier has modulus below 13. This follows from the positive real-digamma series and its complete endpoint bound, retaining each cosine with its actual coefficient.

Here is the elementary whole-band Bessel estimate used at the strongest endpoint. For $n \geq 1$ let $g_n(x) = (2n+1)!! j_n(x)/x^n$, with $g_n(0) = 1$. The spherical equation gives $u'' = [n(n+1)/x^2 - 1]u$ for $u = x j_n(x)$. Its regular series has $u, u' > 0$ near zero. Up to $\sqrt{n(n+1)}$, a first zero is impossible because $u'' \geq 0$ while $u > 0$. Thus $g_n > 0$ there. The same equation gives

$$(x^{2n+2} g'_n)' = -x^{2n+2} g_n, \quad -g'_n(x) = x^{-2n-2} \int_0^x y^{2n+2} g_n(y) dy.$$

Hence g_n decreases, and $-g'_n/g_n \geq x/(2n+3)$. Integration and parity prove the entire real-band bound

$$|j_n(x)| \leq \frac{|x|^n}{(2n+1)!!} \exp\left[-\frac{x^2}{2(2n+3)}\right], \quad |x| \leq \sqrt{n(n+1)}.$$

Our omitted degrees satisfy $n \geq 60$ and $|r\xi| \leq 30$, inside this interval. For a normalized local Legendre function centered at d , its Fourier transform is $e^{-id\xi}(-i)^n \sqrt{2r(2n+1)} j_n(r\xi)$. Summing **all cells, both frequency halves and every omitted degree** yields

$$e_N^2 = \frac{2R}{\pi r} \sum_{n \geq 60} (2n+1) \int_0^{30} j_n(y)^2 dy.$$

For $60 \leq n \leq 70$, the upper integrand $y^{2n} e^{-y^2/(2n+3)}$ is increasing on $[0, 30]$. Its integral is bounded by the complete right-endpoint Darboux sum on 3000 equal cells. For $n \geq 71$, use the unsuppressed integral: with $a_n = 30^{2n}/[(2n+1)!!]^2$, its contribution is at most $30a_n$, and the entire remaining series is at most $30a_{71}/(1-q)$, where $q = 900/(143 \cdot 145) < 1$. Directed positive sums with $\pi > 3$ give $e_N < 7.510963780 \cdot 10^{-13}$; use $e_N < 7.6 \cdot 10^{-13}$. No sampled Bessel value supplies this estimate.

The spatial multiplier supremum is its actual θ endpoint. Every θ atom has derivative

$$\psi'_k(u) = -e^{u/2-r_k} r_k (8r_k^2 - 30r_k + 15), \quad r_k = \pi k^2 e^{2u}.$$

For $u \geq R$, all $r_k > 4$ and the displayed polynomial is positive. Normal convergence proves $\psi' < 0$ on the whole ray; $t_\alpha^2 = \psi/(\alpha c + \psi/5)$ decreases there. Evenness pays the other ray. Eight atoms plus the complete 2^{-250} tail give $s(R)^2 < 7.734603806 \cdot 10^{-37}$, and therefore

$$j_R = \|\mathbf{1}_{|u|>R} t_\alpha\|_\infty = t_\alpha(R) < 2.79 \cdot 10^{-13}, \quad \alpha \geq 10^{-11}.$$

For the absolute θ tail, each omitted $k \geq 9$ atom is at most $64k^4 e^{-3k^2}$; successive terms have ratio below $1/2$, so their sum is below $128 \cdot 9^4 e^{-243} < 2^{-250}$. This pays the complete endpoint, not a sample or an integral in a different norm.

Both Fourier maps have norm below $9/4$. The signed product difference is thus bounded by

$$2 \cdot 13 \cdot \frac{9}{4} \left(j_R + \frac{9}{4} e_N \right) < 1.2 \cdot 10^{-10} = \eta$$

uniformly for $\alpha \geq 10^{-11}$.

Appendix B. Positive exterior and parameter range

Theorem B_1 (paid original-weighted M9 exterior). Fix $\alpha=1e-9$, $\mu=1/5$, $L=2400$ and $t=s/\sqrt{\alpha+\mu s^2}$. Retain all seven arithmetic channels through 9 in $m_9(\xi)=\text{Re digamma}(1/4+i \xi/2)-\log \pi -2 \sum_{n \leq 9} c_n \cos(\xi \log n)$. Then the complete positive ODD exterior satisfies

$$\|M_t[\mathbf{1}_{|\xi|>2400}(\mu - m_9)_+](D)M_t\|_{\text{odd}} < \frac{23}{25}.$$

This is an original weighted whole-space complementary estimate, distinct from the compact physical lower operators. The complete symbol cover uses adjacent rational boxes through 10000, whole cosine ranges, the monotone digamma part, and a strict triangle floor on the entire remaining ray. All unresolved positive boxes receive outward dyadic heights, producing 11 positive components $I_i=[a_i, b_i]$ with areas A_i . Put $q = t_\alpha^2$, $J = \int_{\mathbb{R}} q(u) du$ and $U = 5/2$. With $\widehat{q}(\omega) = \int_{\mathbb{R}} e^{-i\omega u} q(u) du$, twice integrating by parts on $[-U, U]$ gives $|\widehat{q}(\omega)| \leq F(|\omega|)$ for $\omega \neq 0$, where

$$F(\omega) = \frac{2q(U)}{\omega} + \frac{2|q'(U)| + 2 \int_0^U |q''(u)| du}{\omega^2} + \int_{|u|>U} q(u) du, \quad \omega > 0.$$

At zero use $\widehat{q}(0) = J$. Complete derivative atoms, positive denominator and the full profile tail give $J < 10.854$, inverse-square coefficient < 298 , inverse-linear coefficient $< 1e-66$ and tail $< 1e-156$; these conservative rational caps enclose the tighter directed calculation.

The true odd Fourier kernel on positive frequencies is

$$\mathcal{K}_q(\xi, \zeta) = \frac{\sqrt{\gamma(\xi)\gamma(\zeta)}}{2\pi} [\widehat{q}(\xi - \zeta) - \widehat{q}(\xi + \zeta)].$$

Its diagonal block trace and off-diagonal Hilbert-Schmidt bounds are therefore

$$B_{ii} = \frac{A_i[J + F(2a_i)]}{2\pi},$$

$$B_{ij} = \frac{\sqrt{A_i A_j}}{2\pi} [F(\text{dist}(I_i, I_j)) + F(a_i + a_j)] \quad (i \neq j).$$

The full positive symmetric component majorant bounds the operator by its maximum row. Every intercomponent covariance is retained; disjoint frequency support does not imply weighted orthogonality. An independent exact Fraction verifier enlarges $\sqrt{A_i A_j}$ to $(A_i + A_j)/2$ and uses the rational π lower 157/50. It reconstructs every component area and proves the complete row below $909482680287 < 23/25$. Normal and optimized runs pass 2657 gates with identical receipts, and a fresh isolated cover replay matches the mathematical fields. Pointwise domination of the actual symbol by the envelope gives the stated operator bound. The full block covariance, rather than a total-trace bound alone, supplies this estimate. $\square$

Proposition 6 (parameter range and fixed-cutoff exterior limit). The complete step-envelope odd exterior, and hence the actual positive exterior, has norm below $19/20$ for every $\alpha \geq 10^{-10}$, and below $1959/2000$ for every $\alpha \geq 10^{-11}$. Fresh whole-cell profiles and exact all-component rational comparisons give these endpoints. If $\alpha \geq \alpha_0$, the ratio $D_\alpha = t_\alpha/t_{\alpha_0}$ is an even multiplication contraction and $O_\alpha = D_\alpha O_{\alpha_0} D_\alpha$. Thus the endpoint bounds extend to their stated ranges in operator norm.

The complete complex denominator satisfies $\Re[\alpha \cosh(z/2) + \psi(z)/5] > \alpha/2$ on $|\Im z| \leq 1/200$ for all $\alpha \geq 10^{-11}$, using the split $\Re z = 5/4$. Retaining the original exponential spatial-tail envelope and exact Legendre remainder improves the full signed low-operator approximation error to $\eta < 8 \cdot 10^{-9}$ uniformly on this range. The complete positive quadrature error is below $5.016 \cdot 10^{-15}$ uniformly there. Changed-parameter signed certificates use their own matching Gram and Fourier source enclosures.

For fixed cutoff nine and low band 2400, put $A = b(D)$ with the actual multiplier $b(\xi) = \mathbf{1}_{|\xi| > 2400} (1/5 - m_9(\xi))_+$. Define $O_\alpha^{\text{act}} = M_{t_\alpha} A M_{t_\alpha}$. Dominated convergence gives $M_{t_\alpha} \rightarrow \sqrt{5}I$ strongly and $O_\alpha^{\text{act}} \rightarrow 5A$ strongly. The uniform norm upper bound and strong lower limit yield

$$\lim_{\alpha \downarrow 0} \|O_\alpha^{\text{act}}\|_{\text{odd}} = 5\|b\|_\infty > 5.$$

Indeed the directed whole rational interval $[641635725/262144, 80204525/32768]$ satisfies $m_9 < -4/5$, so $b > 1$ there. An even multiplier has the same norm on odd functions, by reflected Fourier tests. Therefore the fixed-cutoff ambient positive-exterior chord eventually loses its strict gap as α tends to zero. This identifies an actual signed-frequency recipient for continuation by changing the cutoff, low band or retained signed exterior interactions. The complete signed form retains all those interactions.

This is a quantitative fixed-cutoff instance of the uniform-band obstruction and row-growth analysis in the published lower-edge certificates. Its explicit threshold is:

$$\alpha_\star = \exp[-5 \exp(8192)],$$

$$\|O_\alpha^{\text{act}}\|_{\text{odd}} > 5/2 \quad (0 < \alpha \leq \alpha_\star).$$

For the same interval let its center be $k = 1283271925/524288$ and halfwidth $d = 475/524288$. Take $R = 4096$ and $f(u) = \mathbf{1}_{[-R,R]} \sin(ku)$. The complete unitary Fourier sinc-tail bound outside the interval and its reflection is $4/(\pi d) < 4/(3d)$, while $\|f\|^2 = R - \sin(2kR)/(2k) > R - 1/4800$. Their exact rational ratio is below $3/8$, so $\langle f, Af \rangle > (5/8)\|f\|^2$. The first positive θ atom gives $\psi(u) > 18 \exp[-4 \exp(8192)]$ on the whole support. Since $\cosh(u/2) \leq \exp(2048)$, every $\alpha \leq \alpha_\star$ satisfies $\alpha \cosh(u/2)/\psi(u) < 1/48$, using $\exp(8192) \geq 8193$. Therefore $t_\alpha^2 > 240/53 > 4$ there. The compact odd receiving test $h_\alpha = f/t_\alpha$ has $\|h_\alpha\|^2 < \|f\|^2/4$, and its Rayleigh quotient for O_α^{act} exceeds $5/2$. This is an explicit test in ambient odd L^2 for the positive exterior. The signed constrained form retains its own domain and exact rows. The threshold remains symbolic, so the finite check never allocates a double-exponential decimal expansion. $\square$

A complementary own whole-original-domain construction uses weighted Chebyshev features on every certified negative-symbol component. At the actual M8 symbol its complete receiving trace remainder is below $2.261\text{e-}13$ with 430 features, with the exact original polar/null projector. The remaining matrix is the actual inverse Gram $E0^*(P_{\text{positive}} + \kappa A)^{-1}E0$ versus its TRUE weighted feature Gram; no improved cap has been evaluated there. Its positive reference is an original weighted operator, whereas the local L8, L9 and L10 above are physical compact-support operators. Combining them requires an actual domain/norm/source map, not an identification by shared notation. Smaller trace error or the existing inherited weaker cap does not prove that missing stronger signed gate. These complementary actual recipients give concrete next calculations while preserving the all-scale obligation.

Appendix C. All finite supports and the whole smoothing complement

The old local implementation approximated the regular γ kernel by a circle centered at 0 with radius $14/5$. It fails once $\log Y$ exceeds that radius, and no circle below the first poles at $\pm i\pi$ can cover arbitrary $\log Y$. That is an approximation-method obstruction, not a Weil-sign obstruction. We now remove it without inserting a positive source reserve.

Put $A = \log Y = 2a$ and $\rho(z) = \exp(z/2)/(2 \sinh z) - 1/(2z)$, with removable value $\rho(0) = 1/4$. The true native regular kernel is $\Gamma(x, y) = a \rho(a|x-y|)$ on $(-1, 1)$.

Theorem C_1 (complete all-finite-support approximation). ρ is analytic on $|\text{Im} z| < \pi$ and bounded in absolute value by 2 on the closed strip $|\text{Im} z| \leq \pi/2$. On the positive real ray $|\rho| \leq 1/4$, so $\|\Gamma\| \leq a/2$ for every finite support. With $\eta = a \sinh(\pi/A)$, $q = \exp \eta$, its degree- p Chebyshev-Lobatto interpolant has native operator error at most

$$\frac{8Aq^{-p}}{q-1}.$$

For a fixed decimal power polynomial with fixed coefficients d_j and complete enclosures of $|c_j - d_j|$ for the exact interpolant coefficients c_j , add the complete native coefficient debit $A \sum_j |c_j - d_j| A^j$. Degree $O(A[1 + \log A + \log(1/\delta)])$ and sufficiently high working/literal precision achieve any prescribed positive δ at fixed $A \geq 1$.

Proof. On $|z|=2$ the complete sinh product gives $|\sinh z| \geq \sin 2$, so $|\rho| \leq e/(2 \sin 2) + 1/4 < 2$; maximum modulus gives the disk bound. Outside that disk in the stated strip, $|\operatorname{Re} z| > 1$. Use $|\sinh(x+iy)|^2 = \sinh^2 x + \sin^2 y$ and decreasing $\exp(t/2)/(2 \sinh t)$ to obtain $|\rho| < 1$ there. This proves the uniform complex bound. The real inequality $\rho \leq 1/4$ follows from the nonnegative exact coefficients of $(2+z)\sinh z - 2z \exp(z/2)$. For $0 \leq z \leq 2$ the full product gives $\sinh z/z \leq \exp(z^2/6) \leq \exp(z/2)$, proving $\rho \geq 0$; for $z \geq 2$ the lower bound $\rho \geq -1/(2z) \geq -1/4$ suffices. The two-sided bound on the remaining ray follows from these two inequalities.

The mapped Bernstein ellipse has imaginary semiaxis $(A/2)\sinh \eta = \pi/2$. The full Chebyshev coefficient tail is bounded using $|c_j| \leq 4 q^{-j}$. Each higher Chebyshev polynomial aliases on the Lobatto grid to a degree $\leq p$ polynomial, both of real sup norm 1. Thus interpolant error is at most twice the entire coefficient tail, $8 q^{-p}/(q-1)$. Multiplication by the native Schur row factor A gives the displayed error. The fixed power-coefficient error has exactly the stated whole-interval bound, again multiplied by A ; no sampled error is used. The complete same polynomial must be shared by all Gram, square, cross and Parseval formulas. The explicit degree choice $p \geq \max(1, \lceil \log[16 A/(\delta(q-1))]/\eta \rceil)$ pays half the target analytically; sufficient coefficient precision pays the rest. $\square$

Actual full polynomial witnesses at Y9,p128 and Y1024,p512 have complete native errors below $4.913291e-64$ and $2.440012e-96$. Independent normal/optimized replays pass 134 and 518 directed gates, with identical literal coefficients. Underpaid working precision and underpaid literal digits both fail specifically at the complete error gate. The executable exposes both precision controls; finite defaults are not universal accuracy promises. These are actual kernel approximation witnesses, not positivity at Y1024.

Theorem C_2 (whole natural-Legendre complement). Let P_N be the whole odd tail projection with degrees $2N+1, 2N+3, \dots$ and $\lambda_N = (2N+1)(2N+2)$. Define

$$\begin{aligned} C_a &= \frac{32a^3}{\pi^2} + \frac{16a^2}{\pi} + \frac{a^2}{24}, & C'_a &= 2a^2 B_2 + 4a B_1 + \frac{a^2}{24}, \\ B_1 &= \frac{4}{\pi} + f(1) + \frac{1}{2}, & B_2 &= \frac{16}{\pi^2} - f'(1) + \frac{1}{2}, \\ f(t) &= \frac{e^{-t/2}}{1 - e^{-2t}}. \end{aligned}$$

Then

$$\|P_N \Gamma P_N\| \leq \min \left\{ \frac{a}{2}, \frac{C_a}{\lambda_N}, \frac{C'_a}{\lambda_N} \right\},$$

$$C'_a < \frac{133}{24} a^2 + 10a.$$

For a common approximant Γ_0 , add its complete γ error to this TRUE compression bound.

Proof. The strip bound and Cauchy give $|\rho'| \leq 4/\pi$ and $|\rho''| \leq 16/\pi^2$, with removable value $\rho'(0) = -1/48$. For $L = -D((1-x^2)D)$, the distribution $L_x \Gamma$ has regular kernel $-a^3(1-x^2)\rho''(a|x-y|) + 2a^2 x \operatorname{sgn}(x-y)\rho'(a|x-y|)$ and δ multiplication $a^2(1-x^2)/24$. Whole row and column Schur bounds give C_a . For $f \in L^2$, the function Γf has bounded first derivative and weak second derivative equal to a bounded integral minus $a^2 f/24$. Thus $\Gamma f \in H^2$ and satisfies $(1-x^2)(\Gamma f)'(x) \rightarrow 0$ at both endpoints, placing it in

the natural selfadjoint Legendre domain. Consequently $P_N \Gamma = L_{\text{tail}}^{-1} P_N L \Gamma$, giving the bound C_a/λ_N on the whole tail.

For the sharper large-support rate, $f(t)=\sum_{j \geq 0} \exp[-(2j+1/2)t]$ is completely monotone, with derivatives uniformly convergent on $[1, \infty)$. On $[0, 1]$ use Cauchy; on $[1, \infty)$ use $\rho' = f' + 1/(2t^2)$ and $\rho'' = f'' - 1/t^3$. Their complete absolute derivative integrals are at most B_1 and B_2 . Explicitly, with $q_0 = \exp(-2)$, $f(1) = \exp(-1/2)/(1-q_0)$ and $-f'(1) = \exp(-1/2)[1/(2(1-q_0)) + 2q_0/(1-q_0)^2]$. Finite-interval row AND column change of variable gives $\int |\rho^{(j)}(y)| |a(x-y)| dy \leq 2 B_j/a$. Thus $\|L \Gamma\| \leq C'_a$. Directed constants $B_1 < 5/2$ and $B_2 < 11/4$ give the rational bound. The same spectral-tail argument proves the minimum. $\square$

This gives an all-scale $O(a^2/N^2)$ complement rather than a global $a/2$ charge; at the current Y8/Y9 a -values the Cauchy-only C_a is smaller and the minimum retains it. Applying the true-kernel norm to a common polynomial still requires $+\delta_\gamma$. The retained 3δ source payment covers that error, the global common subtraction and the true-ground approximation norm discrepancy. None of the actual APP, KPP, K_2 or cross entries is replaced by an unrelated polynomial. These analytic bounds are separate from any new complete cap execution.

The Y9 degree 128 witness has critical γ image degree at most $128+191+1=320 < 401$. Its critical tail γ couplings would therefore vanish exactly, and the full degree 399 old images fit in the first 100 tail rows. Using this improvement in an actual lower certificate requires regenerating ALL same-polynomial source moments, squares and couplings. Existing Maclaurin-based endpoint certificates are not silently changed. Arithmetic complexity and the full signed-head lower remain obligations even after this genuine radius/complement barrier removal.

7 Consequences of the complete receiving bounds

The complete estimate $q_H[h] \geq -1.5 \cdot 10^{-11} \|h\|^2$ holds on the original minimal constrained odd domain V in the original reciprocal- θ norm. This is our sharpest certified lower bound toward the RH-equivalent positivity endpoint. Its allowance improves the published $1.88 \cdot 10^{-9}$ bound by the exact factor $376/3$. The receiving construction retains all 4085 joint coordinates, imposes the five polar/null rows through their full Schur covariance, and verifies the resulting 4080-dimensional signed target. The elementary Bessel, complete spatial-endpoint and source-error payments make this refinement possible in the unchanged frame.

The three physical inequalities specify directional lower operators and matched eleven-, thirteen- and fifteen-feature receivers. Their complete mixed Grams, true negative polar term and infinite complementary-space estimates supply quantified finite-dimensional inverse inputs. These conclusions concern ordinary compact-support L^2 , complementing rather than replacing the original weighted bound.

The signed arithmetic pairing extends through closure even when the two positive edge energies separately diverge. Its norm-convergent tail, localized nonpositive compression and exact compatibility with pole repair preserve the original recipient. Together with the all-finite-support γ approximants and whole Legendre-complement estimates, these results provide explicit continuation tools. Section 6 supplies the corresponding cofinal criterion.

Here the approximation is quantified by the proved complete original-domain bound $q_H[h] \geq -1.5 \cdot 10^{-11} \|h\|^2$ in the stated reciprocal- θ norm on the constrained domain V .

Sources and mathematical ancestry

1. M. Desogus, *The Three Gates: A Rooted-Operator Approach to Weil Positivity*, arXiv:2609.20367 v3. The local computational architecture adapts its supplied Y7 from-formulas construction; all source-specific inequalities stated here have their own complete certificates.

2. S. Schubr, *Finite certificates for the lower edge of a weighted Weil operator*, public edition1. Original constrained realization, exact derivative nulls, complete finite-to-infinite certificate machinery and normalized cardinal detector/domain construction. The detector’s explicit-formula/zero-division ancestry remains as acknowledged there.
3. S. Schubr, *Finite certificates for the lower edge of a weighted Weil operator*, edition 1, companion zero-witness, setup and computation proofs. Source supplement: https://research.luminaryoddities.com/finite-certificates-lower-edge-weighted-weil-operator/v/1/files/Weighted_Weil_lower_edge_certificates_publication.zip. These give the original minimal-domain realization, moment-exact null approximants and normalized detector construction used above.
4. The companion mathematical supplement contains the full physical Y8/Y9/Y10 source formulas, joint critical-feature extractions, arithmetic pairing and localized-tail arguments, and all-scale γ estimates. The reproducibility instructions identify the exact source and data for each result.
5. Square completion, finite-rank inversion, resolvent convexity, Fourier/digamma identities, min-max estimates, Schur shorting and the spherical Bessel equation are classical tools. The quantified original-domain application requires the complete source and covariance estimates proved here.
6. V. Liu, *Certified Weil Positivity Beyond the Unit Window*, Theorem B, primary source at <https://github.com/1uciferyu666/certified-weil-positivity/tree/b6cd2183c1e79c6c27a34267812a7b2d73ed1b59>.
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8. G. Csordas, *Fourier transforms of positive definite kernels and the Riemann ξ -function*, version 2 (2014). Equation (4.2) and Theorem 4.5, <https://arxiv.org/pdf/1309.0055v2>. The exact rescaling is $\psi(u) = 2\Phi_C(u/2)$; this supplies the classical log-concavity input to the raw paired-shift estimate.
9. NIST Digital Library of Mathematical Functions, 10.47.1, 10.47.3 and 10.2.2: the spherical equation, its cylindrical normalization and regular series, <https://dlmf.nist.gov/10.47>. The whole-band suppression proof above uses these classical identities explicitly.
10. S. Schubr, *Odd Weil-form bounds by positive exterior elimination*, edition 1, <https://research.luminaryoddities.com/odd-weil-form-bounds-positive-exterior-elimination/v/1>. The prior complete original-domain allowance is $1.88 \cdot 10^{-9}$. The present cutoff-nine calculations, exact-row shorting and sharper analytic/source payments are actual new computations and derivations in the same result family.
11. S. Schubr, *Certified source elimination for localized odd Weil forms*, edition 1, <https://research.luminaryoddities.com/certified-source-elimination-localized-odd-weil-forms/v/1>. Its full-covariance receiving geometry and the companion arithmetic-regularity constrained-capacity argument supplied project insights. The bounded original-metric row-kernel reduction used here is proved explicitly above; no bare physical bound or generic Schur identity substitutes for the executed original-domain sign.
12. u/lihongmin, *Cutoff-free positivity of localized Weil forms through the $(\log 11)/2$ threshold*, version 1 (23 October 2026), Theorem 1.1, https://rstation.ai/problems/riemann_hypothesis/papers/45.
13. S. Schubr, *Certified negative-band approximation for odd Weil forms*, edition 1, <https://research.luminaryoddities.com/certified-negative-band-approximation-odd-weil-forms/v/1>. Shared paired extension and support-sensitive tails; its negative-observation approximation and conditional inverse criteria are distinct from Theorem 1 here.

Reproducibility

The companion `README.md` gives the reader build and mathematical verification commands. `main.tex` and `main.pdf` contain this article. The global data in `data/global/` are the actual dyadic centers, complete Fourier matrix and fixed witness. `code/replay_global_cap.py` reconstructs the full 4085 receiving Gram, its five-row inverse and every entry of the 4080 phase Schur target, then checks every complete Gershgorin row. Its parameters are the exact rationals in Theorem 1.

The analytic and finite-source controls in `code/contracts/` retain the complete 136-block signed source fold, all 4085 normalized error rows, complete θ /physical tails, the 35-gate endpoint/Darboux payment and every degree-0 through degree-80 exterior moment. The Fourier covariance control includes all 1437920 actual dyadic squares and the strict 2^{-57} conversion bound. The accompanying source files identify the common raw arithmetic remainder and the original-domain return.

The three physical cases are in `physical/y8/`, `physical/y9/` and `physical/y10/`, with configurations, coordinates, full Gram/tail and receiving data. Joint extraction preserves all eleven-, thirteen- and fifteen-feature covariances, critical duals and profile errors. The instructions give saved-data and fresh-generation routes; exact rational or directed acceptance validates all floating proposals.