

# Certified source elimination for localized odd Weil forms

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Luminary Oddities

26 October 2026

## Abstract

We give a quantitative local correction and a form-domain continuation method for the complete odd Weil form. On the support  $|u| < \frac{1}{2} \log 8$  we verify a positive scalar floor together with a  $3 \cdot 10^{-11}$  directional floor along the normalized first-collar polar profile. The sharp coefficient of that profile lies between  $3 \cdot 10^{-11} + 10^{-40}$  and  $5.641396988 \cdot 10^{-11}$ . The lower certificate compresses to every intermediate support between  $\frac{1}{2} \log 7$  and  $\frac{1}{2} \log 8$ . For a compact physical source with a certified gap, we construct an explicit logarithmic Fourier reference and a finite inverse upper using form-core trials and exterior and constraint annihilators. The resulting certificates are complete at every strict finite cap. A rational example exhibits a strict cap certified by the completed receiver although every fixed-extension primal trial fails. Directional inverse metrics, exact polar elimination and portable interval and rational verification provide the local application and its continuation inputs.

## 1 Introduction

**Method and application** The finite inverse-covariance results provide the method; the local collar certificate provides its application. A source block with its poles removed is not the full source block, and a polar profile is not its Fourier symbol. The rational separation example should be read beside the inverse test: it explains which information the receiving inequality requires. The complete local proof remains the independent application of those interfaces.

Eliminating a controlled part of a quadratic form changes the receiving form by its full inverse covariance. A rigorous lower bound must retain that covariance in the original norm. We study this issue for the odd Weil form, including its negative polar term, on bounded physical supports. Our first result is a verified directional replacement at the first collar from  $a_7 = \frac{1}{2} \log 7$  to  $a_8 = \frac{1}{2} \log 8$ . Our second result makes the continuation test accessible to trials from the original form core: an explicit positive Fourier reference is combined with finite corrections in the annihilator of the source space.

The first-collar result is stated after the normalization below. Section 4 gives the form-domain receiver and its strict-cap completeness theorem. Sections 5–6 give the corresponding operator-residual and directional implementations. The appendices provide exact polar elimination, two alternative receiving geometries, a matched quantitative comparison and the local certificate.

The Schur complement, rank-one inversion and functional error majorants are established tools. Desogus [3, Sections 6.11–6.14] already treats the polar-root and harmonic reduction. Our comparison concerns the quantitative receiving coefficient under the explicit physical profile of Section 3. Our earlier weighted realizations [1, 2] provide the original Weil and theta normalization. The larger-window localized certificate of [4] concerns positivity through  $\frac{1}{2} \log 11$ ; the directional coefficient, its matched inverse moment and the error-accounted continuation are the quantities considered here. The form-dual method is a specialization of functional a posteriori estimation [5, 6].

## 2 The physical form and its actual blocks

For real physical tests use the Fourier convention  $\widehat{f}(\xi) = \int_{\mathbb{R}} f(u) e^{-iu\xi} du$  and measure  $d\xi/(2\pi)$ . Write  $\Psi_0 = \Gamma'/\Gamma$ ,  $c_n = \Lambda(n)/\sqrt{n}$ , and

$$Q[f] = \int_{\mathbb{R}} [\Re \Psi_0(1/4 + i\xi/2) - \log \pi] |\widehat{f}(\xi)|^2 \frac{d\xi}{2\pi} - \sum_{n \geq 2} c_n \int_{\mathbb{R}} f(u) [f(u + \log n) + f(u - \log n)] du. \quad (1)$$

On odd tests set  $m(f) = \int f(u) \sinh(u/2) du$ . The full zero-side form is

$$\mathcal{W}[f] = Q[f] - 2m(f)^2. \quad (2)$$

This normalization and its extension to the smooth class whose derivatives decrease faster than every exponential are those of our original weighted realization [1, 2].

Fix  $0 < a < b$ . Decompose the odd physical space into the old interval and the new collar,

$$X = L_{\text{odd}}^2(-a, a), \quad Y = L_{\text{odd}}^2((-b, -a) \cup (a, b)).$$

All embeddings are by zero extension. Denote the actual pole-free blocks by

$$C_b = \begin{pmatrix} A & B \\ B^* & D \end{pmatrix}, \quad p = \sinh(u/2)|_{(-a, a)}, \quad q = \sinh(u/2)|_{\text{collar}}.$$

Thus the full blocks are

$$A_0 = A - 2pp^*, \quad B_0 = B - 2pq^*, \quad D_0 = D - 2qq^*. \quad (3)$$

In particular,  $B_0$  includes the true polar cross term. A symbol such as  $pp^*$  denotes the rank-one map  $x \mapsto p\langle p, x \rangle$ .

These blocks fix the original signed source and receiving forms. The correction addresses the type of harmonic coercivity asserted in [3, Theorem 8.6]. Its derivation below uses the actual form (1)–(2).

**Lemma 1** (Bounded interface and form domain). *The complete cross operator is bounded. With*

$$\beta_b = \frac{\pi}{\sqrt{2}} + 4 + 2 \sum_{2 \leq n < e^{2b}} c_n,$$

*one has*

$$\|B\| \leq \beta_b, \quad \|B_0\| \leq \beta_b + 2\|p\|\|q\|, \quad \|p\|^2 = \sinh a - a, \quad \|q\|^2 = \sinh b - b - \sinh a + a.$$

*The localized logarithmic form has the old/collar direct-sum form domain.*

*Proof.* The digamma representation gives the archimedean cross kernel  $-k(u - v)$ , where

$$k(r) = \frac{e^{-|r|/2}}{1 - e^{-2|r|}} \leq \frac{1}{2|r|} + e^{-|r|/2}.$$

Across one adjoining endpoint the singular part has norm at most  $\pi/2$ . Indeed the kernel  $(x + y)^{-1}$  on the positive half-line has Schur bound  $\pi$  with weight  $x^{-1/2}$ . Combining the two orthogonal collar

inputs gives  $\pi/\sqrt{2}$ . The regular remainder has convolution norm at most  $\int e^{-|r|/2} dr = 4$ . Every arithmetic shift pair has norm at most  $2c_n$ , and shifts of length at least  $2b$  have zero overlap. This proves the cross bound. The polar estimate and the two norms follow directly.

To justify the interface split, start with smooth test functions and remove neighborhoods of width  $\epsilon$  around the two cut points using smooth cutoffs. The removed bounded-amplitude pieces have squared norm  $O(\epsilon)$  and logarithmic Fourier energy  $O(\epsilon \log(1/\epsilon))$ , by dilation of a fixed cutoff. For example the latter bound also follows from Plancherel and the concavity estimate for the logarithmic multiplier using squared derivative norm  $O(\epsilon^{-1})$ . Thus their form norm tends to zero. Arithmetic and polar terms are bounded on the fixed support. Closure, together with the bounded cross operator, yields the asserted split; no boundary trace condition has been imposed.  $\square$

### 3 The verified local application

Write  $a_Y = \frac{1}{2} \log Y$ , and denote by  $A_Y$  the full physical operator of (1)–(2) on odd  $L^2(-a_Y, a_Y)$ , with its localized logarithmic form domain. Put

$$M(7, Y) = \sinh a_Y - \sinh a_7 - (a_Y - a_7), \quad e_Y(u) = \frac{\mathbf{1}_{\{a_7 < |u| < a_Y\}} \sinh(u/2)}{\sqrt{M(7, Y)}}.$$

For  $7 < Y \leq 8$  this is a unit vector. Split  $A_Y$  at  $a_7$ , with old block  $A_7$ , cross block  $B_{7,Y}$  and collar block  $D_{7,Y}$ , and set  $T_{7,Y} = D_{7,Y} - B_{7,Y}^* A_7^{-1} B_{7,Y}$ .

**Theorem 2** (Complete first-collar bound). *Let  $\epsilon = 10^{-40}$  and  $\lambda = 3 \cdot 10^{-11}$ . Then*

$$A_8 \geq \epsilon I + \lambda |e_8\rangle\langle e_8|.$$

*For every  $7 < Y \leq 8$ , zero-extension compression and exact old-variable minimization give*

$$A_Y \geq \epsilon I + \lambda \frac{M(7, Y)}{M(7, 8)} |e_Y\rangle\langle e_Y|, \quad (4)$$

$$T_{7,Y} \geq \epsilon I + \lambda \frac{M(7, Y)}{M(7, 8)} |e_Y\rangle\langle e_Y|.$$

*For  $T_7 = T_{7,8}$  and  $e = e_8$ , the sharp coefficient satisfies*

$$\lambda + \epsilon \leq c_7^{\text{sharp}} := \sup\{c : T_7 \geq c|e\rangle\langle e|\} < 5.641396988 \cdot 10^{-11}. \quad (5)$$

*If  $J$  is the collar inclusion in the full support, then*

$$(5.641396988 \cdot 10^{-11})^{-1} < \langle Je, A_8^{-1} Je \rangle \leq (\lambda + \epsilon)^{-1}. \quad (6)$$

*Proof.* Appendix E gives the complete finite-to-infinite certificate for the first assertion: the true negative polar term, every support-active arithmetic shift, the full critical and robust covariance, and the infinite complement are retained. The portable sources regenerate the analytic intervals and two finite Schur eliminations. The positive shift  $\epsilon$  is subtracted on the whole operator before acceptance.

Compression of  $e_8$  to  $(-a_Y, a_Y)$  is  $\sqrt{M(7, Y)/M(7, 8)} e_Y$ , which proves (4). For a collar vector  $v$ , the full harmonic extension is  $(-A_7^{-1} B_{7,Y} v, v)$ ; its squared norm is at least  $\|v\|^2$ , and its pairing with  $e_Y$  equals that of  $v$ . This gives the stated bound for  $T_{7,Y}$ .

The fixed 200-mode polynomial trial supplied with the certificate has complete original energy divided by its squared literal collar moment strictly below  $5.641396988 \cdot 10^{-11}$ . Schur minimization

bounds  $c_7^{\text{sharp}}$  by that quotient. The lower bound follows from  $\epsilon I + \lambda|e\rangle\langle e| \geq (\epsilon + \lambda)|e\rangle\langle e|$ . Finally  $c_7^{\text{sharp}} = 1/\langle e, T_7^{-1}e \rangle$  by positive rank-one factorization, and block inversion gives  $J^* A_8^{-1} J = T_7^{-1}$ . Taking reciprocals proves (6).  $\square$

The local bound therefore supplies both a certified source gap and a specified improved inverse direction for a later receiving inequality. The actual receiving cross map determines the residual-profile overlaps. Appendix D gives a separate upper consistency test for the prescribed quantitative coefficient under this physical profile.

## 4 A form-core continuation receiver

Let  $X$  be the real odd physical space supported in  $I = (-a, a)$ , optionally restricted by finitely many exact  $L^2$  moment rows. Let  $V$  be its original localized form domain. In this section  $A$  denotes the full old source, including its polar term. Suppose  $A$  obeys  $A \geq gI$ ,  $g > 0$ . With

$$p(\xi) = \Re \Psi_0(1/4 + i\xi/2) - \Psi_0(1/4) \geq 0, \quad m_0 = \Psi_0(1/4) - \log \pi,$$

write  $A = P + K$ , where  $P$  is the compressed  $p(D)$  form and  $K$  is bounded. An explicit full bound is

$$\|K\| \leq b := |m_0| + 2 \sum_{2 \leq n < e^{2a}} c_n + 2(\sinh a - a).$$

Define

$$t = \frac{g}{2(b+g)}, \quad \alpha = \frac{g}{2}, \quad M_X = tP + \alpha I. \quad (7)$$

Convex combination gives  $M_X \leq A \leq CM_X$  on the same domain, where  $C = \max\{1/t, b/\alpha\}$ . On the whole odd line put  $M_{\text{amb}} = tp(D) + \alpha I$  and use the dual norm

$$\|r\|_*^2 = \int_{\mathbb{R}} \frac{|\widehat{r}(\xi)|^2}{tp(\xi) + \alpha} \frac{d\xi}{2\pi}. \quad (8)$$

The expression is the Fourier dual pairing when  $r$  is a distribution.

### 4.1 Keeping the certified direction in the form reference

If the complete source certificate supplies the stronger bound  $A \geq gI + \lambda|e\rangle\langle e|$ ,  $\|e\| = 1$ , the same convex combination gives

$$A \geq tP + \alpha I + \beta|e\rangle\langle e|, \quad \beta = (1-t)\lambda.$$

Let  $Q = tp(D) + \alpha I$  on the ambient odd line and set  $\widetilde{M}_{\text{amb}} = Q + \beta|e^0\rangle\langle e^0|$ . Its restricted form lies below  $A$  and has the same form domain and equivalent norm. The ambient inverse has the explicit dual matrix formula

$$r^* \widetilde{M}_{\text{amb}}^{-1} r = r^* Q^{-1} r - \frac{\beta}{1 + \beta\langle e^0, Q^{-1} e^0 \rangle} (r^* Q^{-1} e^0)((e^0)^* Q^{-1} r). \quad (9)$$

Indeed the reference lower follows from  $(1-t)(gI + \lambda ee^*) + t(P - bI)$ , and direct multiplication proves the rank-one inverse formula. All Fourier moments in (9) are complete integrals in the physical norm. The theorems below also hold with  $\widetilde{M}_{\text{amb}}$  in place of  $M_{\text{amb}}$ : the lower and upper form comparisons persist, and equivalent dual norms preserve the annihilator density. Thus the same finite form receiver retains the verified local direction through its full mixed covariance.

For a profile approximation  $e_a$  with  $\|e - e_a\| \leq \eta$ , let  $\delta_e = (\beta/\alpha)\eta(2 + \eta)$  and  $v_a = (e_a^0)^* Q^{-1} r$ . A safe complete upper, including dual residuals, is

$$r^* \widetilde{M}_{\text{amb}}^{-1} r \leq (1 + \delta_e) r^* Q^{-1} r - \frac{\beta v_a^* v_a}{1 + \beta \langle e_a^0, Q^{-1} e_a^0 \rangle}.$$

To see this, whiten the two rank-one updates by  $Q^{-1/2}$ . Their difference has norm at most  $\delta_e$ , while both inverses of the whitened positive updates have norm at most one. The resolvent identity bounds their inverse difference by  $\delta_e I$ . This charges profile error in the dual metric used for the whole covariance.

For finite columns  $Y$ , write  $a[Y]_{ij} = \mathbf{a}(Y_i, Y_j)$ , where  $\mathbf{a}$  is the closed form of the full source  $A$  in this section.

**Theorem 3** (Annihilator-completed inverse upper). *Let  $E : \mathbb{R}^r \rightarrow X$  have finitely many columns, and let  $Y : \mathbb{R}^r \rightarrow V$ . Extend the restricted residual  $E - A_{\text{form}} Y$  to the whole line by*

$$r_{\text{ext}} = E^0 - p(D)Y^0 - (KY)^0,$$

where the superscript denotes zero extension. If  $\zeta$  annihilates  $V$ , then the full matrix

$$U(Y, \zeta) = E^* Y + Y^* E - a[Y] + (r_{\text{ext}} + \zeta)^* M_{\text{amb}}^{-1} (r_{\text{ext}} + \zeta) \quad (10)$$

satisfies  $E^* A^{-1} E \leq U(Y, \zeta)$ . In particular,  $U(Y, \zeta) \leq D$  proves the finite receiving cap  $E^* A^{-1} E \leq D$ . All mixed dual integrals are included in the matrix.

*Proof.* The form isomorphism  $A_{\text{form}} : V \rightarrow V^*$  and inverse square completion give

$$E^* A^{-1} E = E^* Y + Y^* E - a[Y] + r_X^* A_{\text{form}}^{-1} r_X.$$

Since  $A \geq M_X$ , the last term is at most  $r_X^* M_X^{-1} r_X$ . Its variational supremum ranges over  $V$ , a closed subspace of the ambient form space. Every annihilator completion has the same restriction, so that supremum is at most the ambient dual square in (10). This applies to every source combination, and hence to the complete matrix. The terms are defined for  $Y \in V$ :  $p^2/(tp + \alpha) \leq p/t$  proves that  $p(D)Y^0$  lies in the ambient dual.  $\square$

## 4.2 Finite primal and dual enrichment

Choose form-trial columns  $Z$  and annihilator columns  $W$ . Set

$$A_{\text{ext}} Z = p(D)Z^0 + (KZ)^0, \quad V_0 = [-A_{\text{ext}} Z, W], \quad D_0 = (E^0)^* M_{\text{amb}}^{-1} E^0,$$

and define the full finite matrices

$$\begin{aligned} H &= V_0^* M_{\text{amb}}^{-1} V_0 - \text{diag}(a[Z], 0), \\ N &= \begin{bmatrix} (A_{\text{ext}} Z)^* M_{\text{amb}}^{-1} E^0 - Z^* E \\ -W^* M_{\text{amb}}^{-1} E^0 \end{bmatrix}. \end{aligned} \quad (11)$$

For  $J_0 = [C_0; F_0]$ , formula (10) at  $Y = ZC_0$ ,  $\zeta = WF_0$  equals  $D_0 - J_0^* N - N^* J_0 + J_0^* H J_0$ . Applying Theorem 3 with zero source proves  $H \geq 0$ . If  $c \in \ker H$  and  $c^* N \neq 0$ , scaling that direction makes the displayed upper arbitrarily negative on a source vector, contradicting its fixed nonnegative inverse lower. Thus  $\text{ran } N \subseteq \text{ran } H$ . Square completion and, respectively, a ridge trial give

$$E^* A^{-1} E \leq D_0 - N^* H^\dagger N \leq D_0 - N^* (H + \rho I)^{-1} N, \quad \rho > 0. \quad (12)$$

For the ridge choice the exact trial upper is the last expression minus  $\rho N^* (H + \rho I)^{-2} N$ , so its deletion preserves an upper. Dependent trial columns are admitted by this range argument.

**Theorem 4** (Strict finite-cap completeness). *Suppose nested primal dictionaries are form dense in  $V$  and nested dual dictionaries are dense in the annihilator of  $V$  under (8). For fixed  $A, E, D$ , if  $D - E^*A^{-1}E \geq \eta I_r$  with  $\eta > 0$ , then a finite unregularized receiver in (12) certifies this cap. Its optimized uppers decrease to  $E^*A^{-1}E$  in matrix norm.*

*Proof.* The span of smooth odd functions on the exterior of  $I$ , together with the exact interior moment rows, is dense in the full annihilator. Indeed its preannihilator consists exactly of ambient form functions supported in  $I$  and satisfying those rows, which is  $V$  by the support/core lemma. The Hilbert double-annihilator theorem proves density. Consequently

$$\|r_X\|_{M_X^{-1}}^2 = \inf_{\zeta \in V_{\text{ann}}} \|r_{\text{ext}} + \zeta\|_*^2.$$

Choose form-core  $Y_n \rightarrow A^{-1}E$  columnwise. The reference comparison gives

$$\|A_{\text{form}}(A^{-1}E - Y_n)\|_{M_X^{-1}}^2 \leq Ca[A^{-1}E - Y_n] \rightarrow 0.$$

Finite annihilator completions approximate the dual infimum. Subtracting the exact inverse square from their trial uppers gives a nonnegative matrix whose trace tends to zero, hence whose norm tends to zero. Optimized nested uppers lie between that trial upper and the exact debit, and eventually their error is below  $\eta$ .  $\square$

The theorem concerns a finite source map in the specified compact physical geometry. Complete Fourier tails and numerical covariance errors enter the upper in (10) before its finite sign is accepted.

### 4.3 Why the dual completion changes acceptance

An exact rational example demonstrates the role of exterior columns. Let the ambient logarithmic reference be modeled by

$$P = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}, \quad X = \text{span}(e_1), \quad K_X = -1, \quad A_X = 1, \quad g = \frac{1}{2}, \quad b = 1.$$

Then  $t = 1/6$ ,  $\alpha = 1/4$  and  $M_{\text{amb}} = \begin{pmatrix} 7/12 & 1/6 \\ 1/6 & 7/12 \end{pmatrix}$ . For source  $E = e_1$ , the exact inverse value is 1. All primal trials with the canonical fixed extension have best upper 9/5. Adding  $W = e_2$  gives

$$D_0 = \frac{28}{15}, \quad H = \begin{pmatrix} 5/3 & -4/3 \\ -4/3 & 28/15 \end{pmatrix}, \quad N = \begin{pmatrix} 1/3 \\ 8/15 \end{pmatrix}, \quad D_0 - N^*H^{-1}N = 1.$$

Thus the true strict receiving cap  $D = 3/2$  is certified by the completed receiver and fails every fixed-extension primal trial. The accompanying rational checker verifies these identities exactly.

## 5 Operator-residual implementation

**Theorem 5** (Complete source debit). *Suppose the actual old full block satisfies  $A_0 \geq g_0 I$ ,  $g_0 > 0$ . Let  $Y_0 : Y \rightarrow X$  be a bounded trial lifting with range in  $\text{Dom } A_0$  and  $A_0 Y_0$  bounded. Set*

$$R = B_0 - A_0 Y_0, \quad G_{Y_0} = D_0 - B_0^* Y_0 - Y_0^* B_0 + Y_0^* A_0 Y_0.$$

*On the actual collar form domain,*

$$\boxed{T_W = G_{Y_0} - R^* A_0^{-1} R \geq G_{Y_0} - g_0^{-1} R^* R.} \tag{13}$$

In particular, complete estimates  $G_{Y_0} \geq \gamma I$  and  $R^*R \leq r^2 I$  give

$$T_W \geq (\gamma - r^2/g_0)I.$$

*Proof.* Expansion gives

$$R^*A_0^{-1}R = B_0^*A_0^{-1}B_0 - B_0^*Y_0 - Y_0^*B_0 + Y_0^*A_0Y_0.$$

Subtracting this expression from  $G_{Y_0}$  proves the identity. The lower inequality uses  $0 < A_0^{-1} \leq g_0^{-1}I$ . All displayed corrections to  $D_0$  are bounded, so the identity is a form identity on its stated domain.  $\square$

An approximate source solve is sufficient to propose  $Y_0$ . Acceptance requires its *whole* residual covariance and a complete old-operator lower bound. A finite LDL pivot alone is not such a bound without the basis metric and finite-to-infinite return. Moreover  $G_{Y_0}$  alone is an upper approximation to the Schur form:

$$G_{Y_0} - T_W = R^*A_0^{-1}R \geq 0.$$

It cannot serve as a lower bound without the residual debit. Finite source observations use the complete covariance of their selected source map.

## 6 Directional inverse implementation

Suppose a complete old-block certificate supplies

$$A_0 \geq \mathfrak{M} := \epsilon I + \lambda |e\rangle\langle e|, \quad \epsilon > 0, \quad \lambda \geq 0, \quad \|e\| = 1.$$

The direction and its coefficient must come from that certificate in the same physical metric. Write  $P_e = |e\rangle\langle e|$  and  $Q_e = I - P_e$ .

**Theorem 6** (Directional residual payment). *Without a commutation assumption,*

$$A_0^{-1} \leq \epsilon^{-1}Q_e + (\epsilon + \lambda)^{-1}P_e.$$

*Consequently the residual in Theorem 5 satisfies*

$$T_W \geq G_{Y_0} - \epsilon^{-1}(Q_e R)^*(Q_e R) - (\epsilon + \lambda)^{-1}(P_e R)^*(P_e R). \quad (14)$$

*Proof.* Positive inverse order gives  $A_0^{-1} \leq \mathfrak{M}^{-1}$ . The reference has the displayed inverse on its two orthogonal ranges. Substitute in the exact residual identity (13).  $\square$

For example, the complete localized ground certificate (Theorem 2) supplies

$$A_8 \geq 10^{-40}I + 3 \cdot 10^{-11}|e_8\rangle\langle e_8|,$$

where  $e_8$  is the normalized physical polar profile on  $a_7 < |u| < a_8$ . If the eliminated old block is  $A_8$ , its inverse debit on this direction has coefficient  $10^{40}/(3 \cdot 10^{29} + 1)$  instead of  $10^{40}$ . The orthogonal coefficient remains  $10^{40}$ . The improvement acts on the displayed direction; the scalar floor remains  $10^{-40}$ . Compression to  $A_7$  removes this particular rank-one term because  $e_8$  vanishes on the old interval.

The profile can be realized with a complete error payment. If  $\|e - e_a\| \leq \eta_e$ , put  $c = \lambda/[\epsilon(\epsilon + \lambda)]$ . Then

$$R^*A_0^{-1}R \leq \epsilon^{-1}R^*R - c(R^*e_a)(e_a^*R) + c\|R\|^2\eta_e(2 + \eta_e)I.$$

This follows from  $\|P_e - e_a e_a^*\| \leq \eta_e(2 + \eta_e)$ . The new residual-profile overlaps still require their own full enclosures.

## Further inverse improvement in the actual reference metric

The completed residual method admits a finite trial improvement while preserving the directional reference. This is a square-completion argument; all mixed moments are actual ones of  $A_0$  and  $\mathfrak{M}$ .

**Theorem 7** (Metric-aware residual enrichment). *Let  $Z$  have finitely many columns in  $\text{Dom } A_0$  and define*

$$H_Z = (A_0 Z)^* \mathfrak{M}^{-1} (A_0 Z) - Z^* A_0 Z, \quad N_Z = (A_0 Z)^* \mathfrak{M}^{-1} R - Z^* R.$$

*Then  $H_Z \geq 0$ ,  $\text{ran } N_Z \subseteq \text{ran } H_Z$ , and for every  $\rho > 0$ ,*

$$\begin{aligned} R^* A_0^{-1} R &\leq R^* \mathfrak{M}^{-1} R - N_Z^* H_Z^\dagger N_Z \\ &\leq R^* \mathfrak{M}^{-1} R - N_Z^* (H_Z + \rho I)^{-1} N_Z \leq R^* \mathfrak{M}^{-1} R. \end{aligned} \tag{15}$$

*Here  $H_Z^\dagger$  is the finite Moore–Penrose inverse. No  $\text{Dom}(A_0^2)$  assumption is made.*

*Proof.* For any finite coefficient map  $C$ , inverse square completion at  $ZC$  and  $A_0^{-1} \leq \mathfrak{M}^{-1}$  give

$$R^* A_0^{-1} R \leq R^* \mathfrak{M}^{-1} R - C^* N_Z - N_Z^* C + C^* H_Z C.$$

Positivity follows from the identity

$$H_Z = (A_0 Z - \mathfrak{M} Z)^* \mathfrak{M}^{-1} (A_0 Z - \mathfrak{M} Z) + Z^* (A_0 - \mathfrak{M}) Z \geq 0.$$

If  $H_Z c = 0$ , the first term gives  $A_0 Z c = \mathfrak{M} Z c$ , and hence  $c^* N_Z = 0$ . This proves the range condition. Minimizing the finite square with  $C = H_Z^\dagger N_Z$  proves the first upper bound. The ridge bound follows by choosing  $C = (H_Z + \rho I)^{-1} N_Z$  and discarding the additional nonnegative ridge improvement. All pairings use only  $Z$  and  $A_0 Z$ .  $\square$

Substitution of (15) into (13) yields a further nonnegative receiving improvement. Unlike a restored reference term, it is derived from the entire source inverse and its exact residual. Certified finite moment and profile errors must be charged before the improvement is used numerically.

## Exact return to the weighted compact geometry

Let  $S$  be multiplication by  $s_\theta$  on a fixed compact interval. It is boundedly invertible there. If  $W = S A_0 S$ , then

$$W^{-1} \leq S^{-1} [\epsilon^{-1} Q_e + (\epsilon + \lambda)^{-1} P_e] S^{-1}.$$

For a weighted residual  $R_w$ , the projections in the debit therefore act on  $S^{-1} R_w$ , in that order. They cannot be commuted past  $S^{-1}$ . The baseline in this norm is  $\epsilon S^2$ , not  $\epsilon I$ . On an exact constraint subspace with dense restricted form domain, the inverse of the restricted positive form is at most the compression of the ambient inverse: its variational supremum has fewer test vectors. Thus the ambient bound remains a safe upper after restriction, without identifying the two inverses or dropping a polar row.

## 7 Continuation from the local bound

For a later source equal to the certified  $A_8$ , Theorem 2 supplies  $g = 10^{-40}$  in (7) and the sharper direction with  $\lambda = 3 \cdot 10^{-11}$  in (9) and Theorem 6. The source map and receiving metric are then taken from the actual new block or a completely reduced finite cap. Theorem 3 accepts form-core

trials with explicit exterior corrections; Theorem 7 accepts operator-domain trials with the paid directional metric. In either formulation a certified upper  $U$  and an actual receiving lower  $D$  with  $D - U \geq 0$  prove the corresponding finite receiving inequality. The exact Schur identity then returns it to the original physical form. This gives a concrete continuation test with all source, norm and covariance terms specified.

## A The root functional after elimination

**Theorem 8** (Exact polar-root correction). *Assume  $A \geq gI$ ,  $g > 0$ , and put*

$$\kappa = \langle p, A^{-1}p \rangle, \quad \delta = 1 - 2\kappa > 0, \quad S_C = D - B^*A^{-1}B, \quad \zeta = q - B^*A^{-1}p.$$

*Then  $A_0 \geq \delta A > 0$  and the exact full Schur form is*

$$T_{\mathcal{W}} = D_0 - B_0^*A_0^{-1}B_0 = S_C - \frac{2}{\delta}\zeta\zeta^*. \quad (16)$$

*Consequently  $\mathcal{W}_b \geq 0$  is equivalent to  $T_{\mathcal{W}} \geq 0$  under these old-block hypotheses.*

*Proof.* Write the old coordinate as  $x = v - A^{-1}Bf$ . The pole-free form becomes  $\langle v, Av \rangle + S_C[f]$ , and the polar functional becomes  $\langle p, v \rangle + \langle \zeta, f \rangle$ . Minimizing over  $v$  therefore subtracts

$$2|\langle \zeta, f \rangle|^2 + 4\langle p, A_0^{-1}p \rangle |\langle \zeta, f \rangle|^2 = \frac{2}{\delta}|\langle \zeta, f \rangle|^2,$$

since  $\langle p, A_0^{-1}p \rangle = \kappa/\delta$ . The inequality  $A_0 \geq \delta A$  follows from Cauchy–Schwarz in the  $A$  metric. This proves (16) on the collar form domain.  $\square$

The functional  $\zeta$  is the harmonic pullback of the physical polar row. In general it is not the original collar profile  $q$ . If  $A_0 \geq g_0I > 0$  is already certified, then  $A \geq g_0I$  and

$$\delta = \frac{1}{1 + 2\langle p, A_0^{-1}p \rangle} > 0.$$

This expresses the polar coefficient in the certified old-block metric.

## B A compact-support receiver with zero reference loss

This alternate formulation uses our signed reference construction [2], localized before choosing the reference. Put

$$m_M(\xi) = \Re \Psi_0(1/4 + i\xi/2) - \log \pi - 2 \sum_{2 \leq n \leq M} c_n \cos(\xi \log n).$$

Fix  $R > 0$ , choose an integer  $M \geq e^{2R}$ , and let  $f$  be supported in  $I_R = (-R, R)$ . The omitted arithmetic overlaps are then exactly zero. Choose  $\mu > 0$  and a complete exterior floor  $m_M(\xi) \geq \mu$  for  $|\xi| \geq L$ . Define, on real odd  $L^2(I_R)$ ,

$$b_L = \mathbf{1}_{[-L, L]}(\mu - m_M), \quad K_{R, L} = \mu^{-1}P_R b_L(D)P_R.$$

Here  $D = -i d/du$  in Fourier notation and  $P_R$  includes zero extension.

**Theorem 9** (Support-matched signed bound). *For every compact smooth real odd  $f$  in  $I_R$ ,*

$$Q[f] \geq \mu(\|f\|^2 - \langle f, K_{R,L} f \rangle).$$

*Consequently a complete cap  $K_{R,L} \leq I$  on*

$$\{f : \langle \mathbf{1}_{I_R} \sinh(u/2), f \rangle = 0\}$$

*proves nonnegativity on that entire constrained support, with no reference or arithmetic-tail allowance.*

*Proof.* The support cancellation gives  $Q[f] = \int m_M |\hat{f}|^2 d\xi / (2\pi)$  exactly. On the exterior,  $\mu - m_M \leq 0$ . Dropping only that nonpositive multiplier from  $\mu\|f\|^2 - Q[f]$  proves the inequality. The row is the exact odd polar moment.  $\square$

The band operator is compact. Symmetric cell polynomials of degree below  $p$  and half-width  $r$  give a finite approximation with complete error at most

$$\frac{2\|b_L\|_\infty}{\mu} \sqrt{\frac{2RL}{\pi}} \frac{(rL)^p}{p!}.$$

Indeed this is twice the band norm times the Hilbert–Schmidt Taylor remainder for the Fourier restriction map. The one exact moment row can be included through a verified constrained Schur gate. The cap itself must be checked; compactness is not a sign argument. In the original weighted variables,  $f = s_\theta h$ ,  $s_\theta^2 = \psi / \cosh(u/2)$ , and the reference variable on the compact support is  $\sqrt{\mu}f$ . Thus this cancellation does not set a noninvertible whole-line regularization to zero.

## C An alternative global positive reference

Let  $\psi$  be the positive even theta kernel normalized by  $\mathcal{M}\psi(z) = \xi(1/2 + z)$ , and put  $s_\theta^2 = \psi / \cosh(u/2)$ . The original real odd space consists of  $h \in L^2(\mathbb{R})$  with  $\int s_\theta h \sinh(u/2) = 0$ . Its minimal form domain is the closure of its compact smooth core in  $\|h\|^2 + \int a |\widehat{s_\theta h}|^2 d\xi / (2\pi)$ , and  $q_H[h] = Q[s_\theta h]$  on that core [1, 2]. One may work on the orthogonal complement  $X_0$  of a finite genuine operator-null subspace; its projection preserves this domain and the form energy. All maps below act on  $X_0$ , and the positive form has domain the original minimal domain intersected with  $X_0$ . Write

$$p_M = (m_M)_+, \quad n_M = (m_M)_-, \quad \mathfrak{p}[h] = \int p_M |\widehat{s_\theta h}|^2 \frac{d\xi}{2\pi}, \quad Ch = \sqrt{n_M} \widehat{s_\theta h}.$$

The complete negative symbol has compact frequency support; the positive symbol retains the entire exterior. Since  $|p_M - a|$  is bounded, where  $a = \Re \Psi_0(1/4 + i\xi/2) - \Psi_0(1/4)$ ,  $\mathfrak{p}$  is closed on the same minimal domain. Denote its nonnegative operator by  $P$ . The operator  $C$  is Hilbert–Schmidt, with ambient squared norm

$$\left( \int n_M(\xi) \frac{d\xi}{2\pi} \right) \left( \int s_\theta(u)^2 du \right).$$

All exact quotient projections are retained in  $C$  and its adjoint.

Choose finite orthonormal band observations with inclusion  $J_r$  and set  $E = C^* J_r$ . A complete residual trace bound  $\tau \geq \|(I - J_r J_r^*) C\|_{\text{HS}}^2$  gives  $0 \leq C^* C - E E^* \leq \tau I$ . Let  $\delta_M$  be a complete arithmetic norm debit.

**Theorem 10** (Finite negative-part cap). *If  $\varepsilon - \delta_M - \tau = \kappa > 0$ , put  $L = P + \kappa I$ . For any finite trial map  $Y_1$  with columns in  $\text{Dom } L$ , set  $\mathcal{R} = E - LY_1$  and*

$$U = E^*Y_1 + Y_1^*E - Y_1^*LY_1 + \kappa^{-1}\mathcal{R}^*\mathcal{R}.$$

*If a complete finite enclosure proves  $U \leq I$ , then  $q_H[h] \geq -\varepsilon\|h\|^2$  on the entire original odd domain after exact null lifting.*

*Proof.* The exact signed decomposition and complete remainder give

$$q_H[h] + \varepsilon\|h\|^2 \geq \mathfrak{p}[h] + \kappa\|h\|^2 - \|E^*h\|^2.$$

The right side is nonnegative if  $E^*L^{-1}E \leq I$ . Inverse square completion gives

$$E^*L^{-1}E = E^*Y_1 + Y_1^*E - Y_1^*LY_1 + \mathcal{R}^*L^{-1}\mathcal{R} \leq U,$$

because  $L \geq \kappa I$ . Exact original nulls have zero mixed form pairings, so orthogonal null lifting preserves the lower bound.  $\square$

This keeps the complete positive exterior in  $P$ . Its acceptance test uses the full residual, not an unevaluated inverse or an assumed positive-exterior contraction. Formula (13) and this theorem are the same source-debit principle with different receiving geometries.

## D A matched quantitative comparison

Here we give a consistency test for the particular quantitative coercivity in [3, Theorem 8.6]. The normalized theta kernel satisfies  $\mathcal{M}\psi(z) = \xi(1/2 + z)$  and  $\mathcal{M}\psi'(z) = -z\xi(1/2 + z)$ . The complete zero-side formula therefore gives  $\mathcal{W}(\psi', v) = 0$  on the smooth class whose derivatives decrease faster than every exponential. Let

$$a_7 = \tfrac{1}{2} \log 7, \quad c = \tfrac{1}{2} \log(31/4), \quad a_8 = \tfrac{1}{2} \log 8.$$

Choose the even cutoff  $\chi = 1$  through  $c$ , zero beyond  $a_8$ , and  $\chi = 1 - 3t^2 + 2t^3$  on the transition,  $t = (|u| - c)/(a_8 - c)$ . Put  $F = \chi\psi'$  and  $G = (1 - \chi)\psi'$ . Exact radical cancellation gives  $\mathcal{W}[F] = \mathcal{W}[G]$ . The convenient  $C^1$  cutoff is returned to smooth compact odd tests by  $H^1$  approximation; the strict inequalities below persist.

For the literal physical polar ground on the collar let

$$e = \frac{\mathbf{1}_{\{a_7 < |u| < a_8\}} \sinh(u/2)}{\sqrt{\sinh a_8 - \sinh a_7 - (a_8 - a_7)}}.$$

A whole-cell Arb calculation at 192 bits, with 12000 panels, encloses the complete theta series, physical tails and all arithmetic channels. The accompanying checker proves

$$\mathcal{W}[F] < 2.961707 \cdot 10^{-12}, \quad |\langle e, F \rangle|^2 > 1.651286 \cdot 10^{-11}, \quad \frac{\text{proved upper for } \mathcal{W}[F]}{\text{proved lower for } |\langle e, F \rangle|^2} < 0.179358.$$

The arithmetic head through 16 is paid in norm; the full remaining weighted arithmetic tail is bounded by  $640 \cdot 17^2/20^{17}$ . The archimedean upper is paid by Plancherel, the digamma series and Jensen's inequality. The entire physical norm, derivative and weighted tails are included.

Granting positivity of the old full block, let  $f$  be the restriction of  $F$  to the collar. Schur minimization gives  $\langle f, T_{\mathcal{W}} f \rangle \leq \mathcal{W}[F]$ . Hence any universal bound

$$\langle f, T_{\mathcal{W}} f \rangle \geq g |\langle e, f \rangle|^2 + \text{nonnegative remainder}$$

must have  $g < 0.179358$  under this declared normalization. The compared scalar in the proposed first step is larger than 2.302. An intended different target functional requires its explicit harmonic pullback and a new matched comparison. The test does not determine a positive corrected value of  $g$ : an upper ceiling cannot serve as a lower bound.

## E A complete localized first-collar certificate

This appendix supplies the local numerical input and its finite-to-infinite return. The localized Legendre and kernel architecture follows [3, Section5.1]; the physical polar coefficient is fixed by (2). All local arithmetic, directional updates and complementary covariance bounds are specified below. Put

$$a = \frac{1}{2} \log 8, \quad a_7 = \frac{1}{2} \log 7, \quad r = a_7/a, \\ P_N = \sinh a - \sinh a_7 - (a - a_7), \quad \varepsilon = 10^{-40}, \quad \mu = 3 \cdot 10^{-11}.$$

Let  $e$  be the unit physical odd vector equal to  $\sinh(u/2)/\sqrt{P_N}$  on  $a_7 < |u| < a$  and zero elsewhere. The unitary map  $f(u) \mapsto \sqrt{a}f(ax)$  sends the complete physical operator to

$$A = H + c_0 I + V - K_\gamma - 2a|s\rangle\langle s| - T, \quad c_0 = -\log a - \log(2\pi) - \gamma_E, \quad (17)$$

where  $s(x) = \sinh(ax/2)$ ,  $V(x) = -\frac{1}{2} \log(1 - x^2)$  and

$$K_\gamma(x, y) = a\rho(a|x - y|), \quad \rho(z) = \frac{e^{z/2}}{2 \sinh z} - \frac{1}{2z}, \quad T = \sum_{n \in \{2, 3, 4, 5, 7\}} \frac{\Lambda(n)}{\sqrt{n}} (S_{\log n/a} + S_{\log n/a}^*).$$

The shifts have zero extension outside  $(-1, 1)$ . On polynomials,  $Hf(x) = \frac{1}{2} \int_{-1}^1 (f(x) - f(y))/|x - y| dy$ ; its unit odd Legendre vectors  $\phi_j = \sqrt{(4j+3)/2} P_{2j+1}$  have eigenvalues  $h_{2j+1} = \sum_{k=1}^{2j+1} k^{-1}$ . Polynomial division and the Legendre recurrence prove this diagonalization. The exact kernel decomposition gives (17) on the smooth core and then on its logarithmic form closure. The endpoint multiplication  $V$  retains its full singularity.

**Theorem 11** (Localized physical certificate). *On the complete odd physical form domain supported in  $(-a, a)$ ,*

$$A_8 \geq \varepsilon I + \mu |e\rangle\langle e|.$$

*Consequently its old/collar Schur form satisfies*

$$T_7 \geq \varepsilon I + \mu |e\rangle\langle e|, \quad 3 \cdot 10^{-11} + 10^{-40} \leq c_7^{\text{sharp}} < 5.641396988 \cdot 10^{-11}.$$

### E.1 Common approximation and whole-operator floors

Use the degree-360 Taylor polynomial  $\rho_{360}$  and the degree-199 odd Taylor polynomial  $s_{199}$ , leaving  $V$  and  $T$  exact. Write the resulting operator as  $A_0 = H + c_0 I + K_0$ . For  $0 \leq z \leq 2a < 3$ , the  $\sinh$  product gives  $\sinh z/z \leq e^{z^2/6} \leq e^{z/2}$ , so  $\rho \geq 0$ . Expanding  $(2+z)\sinh z - 2ze^{z/2}$  gives  $\rho \leq 1/4$ . Thus  $\|K_\gamma\| \leq a/2$  and the full polar norm is  $2(\sinh a - a)$ .

The complete arithmetic norm is below  $21/10$ . Its witness is a strictly positive dyadic step function on 384 equal cells. For  $n = 2, 4$  the normalized shifts are exactly  $2/3, 4/3$ , or 128, 256 cells. Each remaining shift lies strictly between its two certified adjacent cell offsets. The program `certify_y8_arithmetic_norm.py` checks every source cell using the larger possible target-cell weight, with zero outside the support, and proves

$$\sum_n c_n \frac{w(x + \log n/a) + w(x - \log n/a)}{w(x)} < \frac{21}{10}.$$

Weighted Young's inequality on each shift and a change of variables then give  $|\langle f, Tf \rangle| \leq (21/10)\|f\|^2$  on the full space.

Set  $R_0 = 14/5$ ,  $q = 2a/R_0$  and

$$M_{R_0} = \frac{e^{R_0/2}}{2 \sin R_0} + \frac{1}{2R_0}, \quad \delta_\gamma = 2aM_{R_0} \frac{q^{361}}{1-q}.$$

The sinh product gives  $|\sinh z| \geq \sin R_0$  on  $|z| = R_0 < \pi$ ; Cauchy's bound proves the displayed complete gamma error. Put

$$r_s = \frac{(a/2)^{201}}{201!} \frac{1}{1 - (a/2)^2/(202 \cdot 203)}, \quad \delta_p = 2a \left[ 2\sqrt{\sinh a/a - 1} \sqrt{2} r_s + 2r_s^2 \right], \quad \delta = \delta_\gamma + \delta_p.$$

Then  $\|A - A_0\| \leq \delta$ . In normalized coordinates let  $e_0 = \sqrt{a/P_N} s_{199}$  on the collar, with zero extension, and set

$$\eta = \sqrt{2a(1-r)/P_N} r_s, \quad \delta' = \delta + \mu\eta(2 + \eta).$$

Since  $\|e - e_0\| \leq \eta$ , it suffices to prove positivity of

$$H + c_0 I + K_0 - \mu e_0 e_0^* - (\delta' + \varepsilon) I. \tag{18}$$

For every tail index  $j \geq 200$  a native diagonal lower floor is

$$d_j = h_{2j+1} + c_0 - \frac{21}{10} - \frac{a}{2} - 2(\sinh a - a) - 3\delta' - \mu - \varepsilon. \tag{19}$$

Drop  $V \geq 0$ , apply the complete norm bounds above, and pay the common approximant error to obtain this inequality. The factor 3 safely pays all appearances of that error. The positive Taylor truncation gives  $\|e_0\| \leq 1$ . The sequence  $d_j$  is increasing and its first value is strictly positive in directed arithmetic.

## E.2 Exact moments and full covariance

The first 200 odd modes form the finite old space. The source expands the Legendre polynomials, gamma images and each shifted polynomial. Products are integrated by exact polynomial and logarithmic moments, including  $V^2$  and every cross product. Define

$$K_{PP} = (\langle \phi_i, K_0 \phi_j \rangle), \quad K_{PP}^{(2)} = (\langle K_0 \phi_i, K_0 \phi_j \rangle), \quad A_{PP} = \text{diag}(h_{2j+1} + c_0) + K_{PP}.$$

With  $b_i = \langle \phi_i, e_0 \rangle$ ,  $z_i = \langle K_0 \phi_i, e_0 \rangle$  and  $n_e = \|e_0\|^2$ , the changed recipient has exactly

$$K'_{PP} = K_{PP} - \mu b b^*, \quad (K_{PP}^{(2)})' = K_{PP}^{(2)} - \mu(z b^* + b z^*) + \mu^2 n_e b b^*.$$

Every low and critical tail row receives the matching rank update. On the positive collar all five minus branches are active and all plus branches are inactive; their exact support inequalities are checked before integrating  $z$ .

Let  $Q$  have ten fixed critical coordinate columns supported in the first 80 modes, and let  $R$  contain 190 completing coordinate columns. The complete Gram of  $[Q, R]$  certifies their invertibility. Pay the rows  $j = 200, \dots, 249$  exactly, with denominators  $d_j$ . If their full-basis matrix is  $B$ , the exact Parseval remainder

$$G_{\text{res}} = (K_{PP}^{(2)})' - (K_{PP}')^* K_{PP}' - B^* B$$

is the covariance of the entire remaining tail. Its robust compression, divided by  $d_{250}$ , bounds all remaining robust inverse covariance. The critical rows are computed separately through  $j = 4999$ .

### E.3 The entire infinite critical tail

Critical polynomials have degree at most  $D = 159$ ; their gamma images have degree at most 520, and the polar polynomial has degree 199. For  $n_0 = 10001$  put

$$r_F = (n_0 - D - 2)^{-1}, \quad s_F = \left( \frac{n_0}{n_0 - D - 2} \right)^{3/2}, \quad n_F = \sqrt{\frac{2n_0 + 1}{2n_0}}.$$

Repeated integration by parts uses the exact primitives

$$J_n^{(1)}(t) = \frac{P_{n+1}(t) - P_{n-1}(t)}{2n + 1}, \quad J_n^{(d+1)}(t) = \frac{J_{n+1}^{(d)}(t) - J_{n-1}^{(d)}(t)}{2n + 1}.$$

At each shift cut  $t = 1 - \log n/a$ , the Bernstein bound and the factors  $r_F, s_F, n_F$  enclose the whole derivative endpoint sum. Summing all five arithmetic channels gives constants  $C_{\text{ar},k}$  with a  $C_{\text{ar},k}/n$  bound. The endpoint potential has exact coefficients

$$\langle \phi_n, V \phi_m \rangle = \frac{2\sqrt{(2n+1)/2}\sqrt{(2m+1)/2}}{(n-m)(n+m+1)} \quad (n > m \text{ odd}).$$

Expanding the denominator in  $m(m+1)/(n(n+1))$ , retaining eight signed moments, and bounding the remaining geometric series by  $159 \cdot 160/(10001 \cdot 10002) < 1$  gives constants  $C_{V,k}$  such that

$$|\langle \phi_n, K_0 Q_k \rangle| \leq \frac{C_{\text{ar},k}}{n} + \frac{C_{V,k}}{n^{3/2}} \quad (n \geq 10001 \text{ odd}).$$

The collar profile contributes  $C_e/n$ , calculated by the same primitive sum at  $r$  through degree 199. If  $\beta = Q^*b$ , replace  $C_{\text{ar},k}$  by  $C'_{\text{ar},k} = C_{\text{ar},k} + \mu|\beta_k|C_e$ . For  $J = 5000$  use the complete scalar bounds

$$S_2 = \frac{1}{4(J-1/2)}, \quad S_{5/2} = 2^{-5/2} \left( J^{-5/2} + \frac{2}{3} J^{-3/2} \right), \quad S_3 = \frac{J^{-3} + \frac{1}{2} J^{-2}}{8}.$$

The entire far critical covariance is bounded by the diagonal matrix with entries

$$10 \frac{(C'_{\text{ar},k})^2 S_2 + 2C'_{\text{ar},k} C_{V,k} S_{5/2} + C_{V,k}^2 S_3}{d_{5000}}.$$

Cauchy–Schwarz for ten columns gives the factor 10. Adding the exact rows  $j = 250, \dots, 4999$  gives  $S_{\text{crit}}$ .

## E.4 Finite acceptance and physical return

Subtract  $\delta' + \varepsilon$  from the native old diagonal. Let  $A_U, A_R, A_{LR}$  be its  $Q, Q$ ;  $R, R$ ; and  $R, Q$  blocks, and let  $W_L, W_R, X$  be the first-slab inverse covariance blocks. With  $Y = A_{LR} - X$  and  $\theta = 3/4$ , Young's inequality gives

$$S_R = A_R - W_R - (1 + \theta)R^*G_{\text{res}}R/d_{250}.$$

A proposed finite congruence  $T_P$  is accepted through  $T_P^*S_RT_P \geq gI$ ,  $g = 0.9999999999$ . Consequently  $S_R^{-1} \leq T_PT_P^*/g$ . The final matrix

$$F = A_U - W_L - (1 + 1/\theta)S_{\text{crit}} - Y^*T_PT_P^*Y/g$$

has ten strictly positive interval LDL pivots. The standard-library checker repeats this interval-family test, the robust inverse floor and the old coordinate invertibility with exact outward 300-decimal rational arithmetic. Thus the full operator (18) is positive, which proves the ambient assertion of Theorem 11.

The old compression is bounded below by  $\varepsilon I$ . Its bounded old/collar interface admits the exact harmonic extension  $H_7f = (-A_7^{-1}Bf, f)$ . Since  $e$  is supported in the collar,

$$\langle f, T_7f \rangle = \langle H_7f, A_8H_7f \rangle \geq \varepsilon\|f\|^2 + \mu|\langle e, f \rangle|^2.$$

For the upper endpoint, `Native receiving trial.py` proposes an explicit 200-mode polynomial with exact decimal coefficients and then encloses its complete original energy and literal collar moment. The energy is below  $1.242543790908 \cdot 10^{-29}$  and the squared signal is above  $2.202546272938 \cdot 10^{-19}$ . The full directed quotient is below  $5.641396988 \cdot 10^{-11}$ . Schur minimization proves the stated upper for  $c_7^{\text{sharp}}$ ; the lower follows from  $\varepsilon I + \mu ee^* \geq (\varepsilon + \mu)ee^*$ .

For  $7 < Y \leq 8$ , put  $M(7, Y) = \sinh(a_Y) - \sinh(a_7) - (a_Y - a_7)$  and let  $e_{7,Y}$  be the unit physical collar profile. Zero-extension compression restricts  $e$  to  $\sqrt{M(7, Y)/M(7, 8)}e_{7,Y}$ . Hence

$$A_Y, T_{7,Y} \geq \varepsilon I + \mu \frac{M(7, Y)}{M(7, 8)} |e_{7,Y}\rangle\langle e_{7,Y}|,$$

where the two operators act on their respective full-support and collar spaces. This is the continuum first-collar application.

**Reproduction.** The accompanying sources run with Python 3.12, NumPy 2.3.5 and python-flint 0.9.0. Run `Reproduce corrected first step.py` with an explicit empty `-output-dir` outside the source folder. It constructs primitive moments, complete finite and infinite tails, both shifted certificates, the direct trial and exact-rational normal/optimized checks. All acceptance gates are explicit exceptions and remain active under Python optimization.

## F Reproduction

The directory `local-certificate` contains the source generator, fixed arithmetic witness, configuration and exact final checkers for Theorem 2. Its README gives a single fresh-output command. The generated data include the complete Gram, finite and analytic tail covariances, positive scalar and directional lower certificates, and the fixed-polynomial collar upper. The rational continuation example is reproduced by `code/verify_annihilator_receiver.py`. For the separate comparison in Appendix D, run `python code/verify_first_step.py -panels 12000 -output comparison.json`. Its theta, overlap and norm enclosures are independent of the local positive-certificate generator. Build the manuscript with `tectonic main.tex`.

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