

Certified complementary bounds for an odd Weil receiver

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Abstract

We certify the complete positive exterior of a theta-weighted odd Weil receiver on the orthogonal complement of its finite phase space. For the arithmetic prefix through nine, the exterior beyond frequency 2400 has complementary norm below $1/5$, uniformly for regularization $\alpha \geq 10^{-14}$, on the existing 4080-dimensional odd phase span. A positive-block Schur estimate pays every exterior mixed term through a finite joint covariance and yields an explicit sufficient signed matrix inequality with exact rows retained. The same exterior has a unique ambient norm-one transition in $(4.46, 4.47) 10^{-13}$. A direct-density rule with 352 positive carriers encloses the actual operator in norm; complete root, quadrature, physical-tail and finite-arithmetic bounds certify both conclusions. The argument organizes receiving estimates around the full complement of the phase span and supplies portable reproducing code and data.

1 The complementary estimate and its use

The finite part of a signed operator estimate is useful only together with its entire complementary space and the mixed terms coupling them. For the odd weighted Weil receiver considered here, those estimates can be organized around a finite phase span even when an ambient positive-exterior inverse has lost coercivity. The main result is a uniform bound on that whole complement.

Let $\mathcal{H} = L^2_{\text{odd}}(\mathbb{R}, du)$, with inner products conjugate-linear first and Fourier convention

$$\widehat{f}(\xi) = \int_{\mathbb{R}} f(u) e^{-iu\xi} du, \quad \|f\|^2 = \int_{\mathbb{R}} |\widehat{f}(\xi)|^2 \frac{d\xi}{2\pi}.$$

For $u \geq 0$ put

$$\psi(u) = \sum_{n \geq 1} (4\pi^2 n^4 e^{9u/2} - 6\pi n^2 e^{5u/2}) e^{-\pi n^2 e^{2u}}, \quad s(u)^2 = \frac{\psi(u)}{\cosh(u/2)}, \quad (1)$$

and use the even theta continuation. The weight is smooth, strictly positive and bounded. Write $\Psi = \Gamma'/\Gamma$, $\mu = 1/5$ and

$$m_9(\xi) = \Re \Psi(1/4 + i\xi/2) - \log \pi - 2 \sum_{2 \leq n \leq 9} \frac{\Lambda(n)}{\sqrt{n}} \cos(\xi \log n), \quad (2)$$

$$b(\xi) = 1_{\{|\xi| > 2400\}} (\mu - m_9(\xi))_+, \quad t_\alpha = \frac{s}{\sqrt{\alpha + \mu s^2}}, \quad O_\alpha = M_{t_\alpha} b(D) M_{t_\alpha}. \quad (3)$$

Here Λ is the von Mangoldt function. The nonzero arithmetic channels in (2) are 2, 3, 4, 5, 7, 8, 9. Multipliers use the displayed Fourier normalization. The positive operator O_α acts on $\mathcal{H}$.

Partition $[-17/10, 17/10]$ into 136 cells of halfwidth $r = 1/80$. Let V_α be the odd span of t_α times polynomials of degree at most 59 on each cell, with zero extension outside this interval. Odd reflection leaves 68 independent cells, so $\dim V_\alpha = 4080$. Let Q_α project orthogonally onto $V_\alpha^\perp$ in $\mathcal{H}$.

Theorem 1.1 (Complete complementary bound). *For every $\alpha \geq 10^{-14}$,*

$$\|Q_\alpha O_\alpha Q_\alpha\| < \frac{1}{5}. \quad (4)$$

The full computed operator upper is less than 0.193949577332. Consequently the restriction of $I - O_\alpha$ to $V_\alpha^\perp$ has lower bound greater than $4/5$. The same bound holds after adjoining finitely many odd row profiles to V_α and taking the smaller orthogonal complement.

The proof in Section 4 pays every omitted polynomial degree and the entire physical exterior. A complete positive covariance trace bounds all carrier combinations at once. The projector in (4) is the actual weighted-space projector, rather than an unweighted polynomial projector.

Theorem 2.1 turns (4) into a finite signed receiving condition. Its exterior cross-term cost is governed by the complementary bound $1/5$. The relevance of this choice is visible in a second actual-operator result.

Theorem 1.2 (Ambient threshold). *There is a unique $\alpha_c > 0$ with $\|O_{\alpha_c}\| = 1$, and*

$$4.46 \cdot 10^{-13} < \alpha_c < 4.47 \cdot 10^{-13}.$$

At the two endpoints the respective certified bounds are

$$\|O_{4.46 \cdot 10^{-13}}\| > 1.000000999, \quad \|O_{4.47 \cdot 10^{-13}}\| < 0.999999001.$$

Thus the ambient recipient $aI - O_\alpha$ fails strict positivity for $a \leq 1$ and $0 < \alpha \leq 4.46 \cdot 10^{-13}$. The complete complementary estimate (4) remains applicable throughout $\alpha \geq 10^{-14}$. Section 5 proves the threshold and its uniqueness.

The phase frame and original-domain receiving construction descend from the weighted-Weil certificate program [1, 2]; the residual-covariance viewpoint also appears in [3]. Schur reduction, positive block factorization and finite covariance estimates are classical [4, 5]. The source-specific contributions here are the complete actual-density construction, its complementary estimate on this existing phase span, the certified ambient threshold and their joint finite receiving application. The spherical-Bessel identities are standard [6].

2 A signed receiving condition with all mixed terms

The following bounded-operator statement identifies precisely how to use the complementary estimate. It also accommodates dependent row or phase columns.

Theorem 2.1 (Positive exterior Schur reduction). *Let $O \succeq 0$ be bounded on a Hilbert space. Let $Z : \mathbb{C}^d \rightarrow \mathcal{H}$ have finite-dimensional range, containing the columns of an exact row map $\mathcal{R}$. Put $P = \text{Proj ran } Z$, $Q = I - P$, and suppose $QOQ \preceq \tau Q$ with $a > \tau$. Let $C = C^*$ and set $K_0 = ZCZ^*$. Suppose*

$$\|O - UU^*\| \leq \delta, \quad G = Z^*Z, \quad F = U^*Z.$$

If

$$aG - GCG - \frac{a}{a - \tau}(F^*F + \delta G) \succeq 0 \quad \text{on } \{y : \mathcal{R}^*Zy = 0\}, \quad (5)$$

then $aI - K_0 - O \succeq 0$ on $\ker \mathcal{R}^*$. For $O = O_\alpha$ and a span containing V_α , Theorem 1.1 supplies $\tau = 1/5$ uniformly for $\alpha \geq 10^{-14}$.

Proof. In $P\mathcal{H} \oplus Q\mathcal{H}$, write

$$O = \begin{pmatrix} A & B \\ B^* & D \end{pmatrix}, \quad 0 \preceq D \preceq \tau I.$$

Positive block factorization gives $B = A^{1/2}TD^{1/2}$ with $\|T\| \leq 1$; bounded regularization proves the same estimates when a block is singular. Spectral calculus implies

$$B(aI - D)^{-1}B^* \preceq \frac{\tau}{a - \tau}A. \quad (6)$$

Since $K_0 = PK_0P$, the Schur complement of $aI - K_0 - O$ is bounded below by

$$aP - K_0 - \frac{a}{a - \tau}POP.$$

For $x = Zy$, its quadratic form is bounded below by the left side of (5), because $Z^*OZ \preceq F^*F + \delta G$. Every vector in $Q\mathcal{H}$ satisfies the exact rows: $\text{ran } \mathcal{R} \subseteq \text{ran } Z$. Hence $\ker \mathcal{R}^*$ decomposes as its finite P part plus all of $Q\mathcal{H}$, and the Schur argument applies on this original row kernel. No inverse of G is needed. $\square$

The inequality (6) pays the complete exterior off-range covariance through a positive operator bound. All phase/row/carrier mixed entries remain in F^*F and G . At $a = 1$ the factor $a/(a - \tau)$ is $5/4$; for a complete low approximation debit η , use $a = 1 - \eta$ and retain every other outside debit in this same scalar budget.

For clarity, the original weighted use can be stated directly. Let $\mathcal{K} = \{h \in L^2_{\text{odd}}(\mathbb{R}) : \langle h, s \sinh(u/2) \rangle = 0\}$ and let V be the closure of $C_c^\infty(\mathbb{R}) \cap \mathcal{K}$ in the graph norm

$$\|h\|^2 + \int_{\mathbb{R}} a(\xi) |\widehat{sh}(\xi)|^2 \frac{d\xi}{2\pi}, \quad a(\xi) = \Re \Psi(1/4 + i\xi/2) - \Psi(1/4) \geq 0.$$

The last sign follows term by term from $a(\xi) = \sum_{n \geq 0} \frac{(\xi/2)^2}{(n+1/4)((n+1/4)^2 + (\xi/2)^2)}$. The arithmetic shifts are bounded on this original Hilbert space, so the retained form has this same minimal domain. Let H_9 be that form with

$$q_{H_9}[h] = \int_{\mathbb{R}} m_9(\xi) |\widehat{sh}(\xi)|^2 \frac{d\xi}{2\pi}.$$

On its original polar-admissible core, put $x = \sqrt{\alpha + \mu s^2} h$. Then $t_\alpha x = sh$, and

$$q_{H_9}[h] + \alpha \|h\|^2 = \|x\|^2 - \int_{\mathbb{R}} (\mu - m_9(\xi)) |\widehat{t_\alpha x}(\xi)|^2 \frac{d\xi}{2\pi}. \quad (7)$$

The original odd polar row becomes $t_\alpha \sinh(u/2)$ in the x variable. On $|\xi| > 2400$, the positive part of $\mu - m_9$ is exactly b . If the complete low-frequency operator has a same-domain finite approximation ZCZ^* with norm error at most η , (7) is bounded below by $\langle x, [(1 - \eta)I - ZCZ^* - O_\alpha]x \rangle$. Thus (5), on the original transformed row kernel, implies $q_{H_9}[h] \geq -\alpha \|h\|^2$. A proved original-norm arithmetic estimate $\|H - H_9\| \leq \delta_9$ then gives $q_H[h] \geq -(\alpha + \delta_9) \|h\|^2$. These implications use the same original core and its specified closure. Additional exact rows enter this return when their original-domain reduction is supplied.

To apply (5) with numerical data, source errors must be transported jointly. For example, if $D_0 \succ 0$,

$$\|(G - \widehat{G})_{D_0}\| \leq e_G, \quad \|(F - \widehat{F})D_0^{-1/2}\| \leq e_F,$$

where the first norm denotes congruence by $D_0^{-1/2}$, then

$$F^*F \preceq \widehat{F}^*\widehat{F} + (2\|\widehat{F}D_0^{-1/2}\|e_F + e_F^2)D_0.$$

The signed products involving G and C are enclosed in that same coefficient metric before testing (5). Separate cross-block errors can be paid by Young's inequality: diagonal errors e_G, e_K and mixed norm e_E admit pads $e_G + \sigma e_E$ and $e_K + e_E/\sigma$ for every $\sigma > 0$.

3 A direct positive rule for the actual exterior

The actual multiplier b in (3) has eleven bounded positive-frequency lobes. A complete interval cover of $[2400, 10000]$ and the monotonic digamma bound beyond 10000 certify that these contain its full positive set. The reader code constructs the cover from (2), including all seven channels and every interval. The dyadic step majorant is used for coverage; the quadrature below integrates the actual analytic density.

Write $d(\xi) = \mu - m_9(\xi)$. On each covered component $I = [l, r]$, direct interval evaluation proves

$$d(l) < 0, \quad d(r) < 0, \quad d((l+r)/2) > 0, \quad d'' < 0 \text{ on } I.$$

The last claim is checked on 16 adjacent full interval subcells, using

$$d''(\xi) = \frac{1}{4} \Re \Psi^{(2)}(1/4 + i\xi/2) - 2 \sum_{2 \leq n \leq 9} \frac{\Lambda(n)(\log n)^2}{\sqrt{n}} \cos(\xi \log n).$$

Strict concavity gives exactly two roots and identifies the entire positive interval between them. Dyadic bisection encloses each root in a bracket of width at most 2^{-60} , with opposite endpoint signs. The inner root-bracket endpoints define $J = [c-h, c+h]$, on which $d > 0$. The two omitted root boxes have positive mass bounded by their exact lengths times a full interval upper for d_+ . The complete root brackets and interval cover are supplied as mathematical data.

Use 32-point Gaussian quadrature for $d(\xi) d\xi$ on each J . Every exact node has $d > 0$. Replace its frequency and positive measure weight by fixed binary64 dyadics ξ_j, w_j , and define

$$u_j(u) = \sqrt{w_j/\pi} t_\alpha(u) \sin(\xi_j u), \quad U_\alpha = [u_1, \dots, u_{352}]. \quad (8)$$

Both the positive and reflected negative frequencies are incorporated in this odd-space normalization. In particular, the odd Fourier multiplier has the rank-one integral representation

$$O_\alpha = \frac{1}{\pi} \int_{2400}^{\infty} d(\xi)_+ |t_\alpha \sin(\xi u)\rangle \langle t_\alpha \sin(\xi u)| d\xi.$$

Proposition 3.1 (Whole-operator quadrature). *The positive literal rule (8) satisfies*

$$\|O_\alpha - U_\alpha U_\alpha^*\| < 7.234 \cdot 10^{-12} < 10^{-9} \quad (\alpha \geq 10^{-14}). \quad (9)$$

Its literal measure mass is less than 1.601.

Proof. The analytic payments are uniform in α on the stated range. Put $q = t_\alpha^2$ and truncate physical variables to $[-L, L]$, $L = 5/2$. Since $q \leq 5$, the complete theta envelope in Appendix A gives

$$\int_{\mathbb{R}} q < 26, \quad T = \int_{|u|>L} q \leq \frac{240}{\alpha} \left(\frac{e^5}{3} + \frac{1}{9} \right) e^{-3e^5} < 10^{-130}. \quad (10)$$

The analytic continuation of d is

$$d(z) = \mu + \log \pi - \frac{\Psi(1/4 + iz/2) + \Psi(1/4 - iz/2)}{2} + 2 \sum_{2 \leq n \leq 9} \frac{\Lambda(n)}{\sqrt{n}} \cos(z \log n).$$

The complex real-part operation is replaced by this symmetric analytic sum. On the parameter-two Bernstein ellipse of every J , full rectangle evaluation gives $|d| < 100$, with the digamma arguments away from their poles. The bilinear continuation of the sine rank-one kernel agrees with its real selfadjoint value. Its operator norm is bounded by its Hilbert–Schmidt norm, at most

$$H = \frac{26}{\pi} 100 \exp(2Lh(2 - 1/2)/2).$$

Gaussian exactness through degree 63 and the positive-rule Chebyshev-tail estimate bound its complete analytic error by $8hH 2^{-64}/(1 - 1/2)$.

If (η_j, ν_j) is the exact Gaussian frequency/measure pair and (ξ_j, w_j) its literal replacement, the truncated kernel K has $\|K\| \leq 26/\pi$ and $\|K'\| \leq 2L 26/\pi$. Thus the entire literal payment is

$$\frac{26}{\pi} \sum_j |\nu_j - w_j| + \frac{2L 26}{\pi} \sum_j \nu_j |\eta_j - \xi_j|.$$

Root-box positive mass costs at most $26/\pi$ times that mass. For a continuous measure of mass A and the discrete rule of mass W , the full physical truncation payment is

$$\frac{A + W}{\pi} (2\sqrt{26T} + T).$$

This follows by expanding each full carrier into its inside and outside parts and bounding the rank-one cross terms. The continuous mass A is enclosed using the same analytic rule and root-box debit. Summation of all eleven components, with outward arithmetic, gives a total less than $7.234 10^{-12}$. All estimates precede restriction to any finite source map, so (9) holds on the whole odd space. $\square$

4 The entire phase complement

Here the complete complementary estimate follows from exact plane-wave polynomial energy rather than from testing complementary basis vectors.

Lemma 4.1 (Full polynomial residual). *For real x let j_k be the spherical Bessel function and put*

$$e_{60}(x) = 1 - \sum_{k=0}^{59} (2k+1) j_k(x)^2.$$

The squared L^2 error of projecting a plane wave of frequency ξ onto degree at most 59 on a cell of halfwidth r is $2r e_{60}(r\xi)$. A real sine of arbitrary cell-center phase admits a real polynomial approximation with no larger error.

Proof. The plane-wave expansion [6, Eq. 10.60.7], together with $\int_{-1}^1 P_k^2 = 2/(2k+1)$, gives

$$\sum_{k \geq 0} (2k+1) j_k(x)^2 = 1.$$

Orthogonality and rescaling give the exact residual. Taking the imaginary part after multiplication by the cell-center unit phase proves the sine assertion. $\square$

Proof of Theorem 1.1. On each negative cell choose the sine approximation of Lemma 4.1; reflect it oddly and multiply by t_α . The resulting function belongs to V_α . Since $t_\alpha^2 \leq 5$, summing the 136 cells gives the squared inside approximation error at most

$$136 \cdot 2r \cdot 5 e_{60}(r\xi) = 17e_{60}(\xi/80).$$

Orthogonal projection minimizes distance in the actual weighted span. Thus every literal carrier obeys

$$\|Q_\alpha u_j\|^2 \leq \frac{w_j}{\pi} \left[17e_{60}(\xi_j/80) + \int_{|u|>17/10} t_\alpha^2 du \right].$$

The positive complete carrier covariance has operator norm at most its trace. Consequently

$$\|Q_\alpha U_\alpha U_\alpha^* Q_\alpha\| \leq \frac{17}{\pi} \sum_j w_j e_{60}(\xi_j/80) + \frac{\sum_j w_j}{\pi} \int_{|u|>17/10} t_\alpha^2 du. \quad (11)$$

This trace controls every linear combination of carriers, including mixed entries. It is a whole-complement bound.

With $X = e^{17/5}$ the full real theta envelope gives

$$\int_{|u|>17/10} t_\alpha^2 du \leq 169 \cdot 10^{14} (X/3 + 1/9) e^{-3X}.$$

Evaluate all 352 literal terms of (11), including all 60 Bessel coefficients, by outward arithmetic, and add 10^{-9} from Proposition 3.1. The resulting certified upper satisfies

$$\text{upper} < 0.193949577332 < \frac{1}{5}. \quad (12)$$

Every omitted frequency and physical region has been paid. Compression of a positive operator to a smaller orthogonal complement cannot increase its norm, proving the row-enlargement assertion. $\square$

The value in (12) is the outward upper produced by the complete computation, rather than a proposed extremal eigenvalue. The reader program evaluates $j_0(x) = \sin x/x$, $j_1(x) = \sin x/x^2 - \cos x/x$ and the finite exact recurrence $j_{k+1} = (2k+1)j_k/x - j_{k-1}$. All rounding is enclosed by Arb.

5 The actual ambient transition

The same positive rule supplies a complete finite carrier Gram $K_\alpha = U_\alpha^* U_\alpha$. All 352 columns and every mixed entry are integrated on the whole line. Appendix B describes its certified outward radius.

Let $\widehat{K}$ be its dyadic center and ε_K its complete operator error. The literal computation proves

$$\widehat{K} - \varepsilon_K I \preceq K_\alpha \preceq \widehat{K} + \varepsilon_K I, \quad \varepsilon_K < 8.724 \cdot 10^{-7}.$$

At $\alpha = 4.47 \cdot 10^{-13}$, an exact dyadic congruence and all Gershgorin rows prove $K_\alpha \prec 0.999999I$. At $\alpha = 4.46 \cdot 10^{-13}$, an exact dyadic Rayleigh vector proves $\|K_\alpha\| > 1.000001$. Since $\|U_\alpha U_\alpha^*\| = \|K_\alpha\|$, Proposition 3.1 gives the endpoint bounds of Theorem 1.2.

For uniqueness, put $S = \|s\|_\infty^2 < \infty$. If $\alpha > \beta > 0$, then

$$O_\alpha = D O_\beta D, \quad D = M_{t_\alpha/t_\beta}, \quad \|D\|^2 \leq \frac{\beta + \mu S}{\alpha + \mu S} < 1.$$

The multiplier is nonzero, so $\|O_\beta\| > 0$. Hence the norm is strictly decreasing. The explicit formula for t_α makes it continuous in multiplication-operator norm on compact positive parameter intervals; therefore $\|O_\alpha\|$ is continuous. The two certified endpoints give existence and uniqueness. This completes the proof of Theorem 1.2.

6 Organization of a complete receiving computation

Theorem 1.1 supplies a complementary prerequisite for (5) on an existing finite span. Theorem 1.2 identifies a parameter interval where an ambient elimination would have a different applicability boundary. A signed receiving computation can retain the complete phase, exact-row and exterior covariance in either of two finite organizations.

The first is (5), with the exterior mixed Schur term paid by (6). The second adjoins the 352 carrier columns to the 4080 phase columns and the five exact rows of the companion receiver. Its 4437-coordinate joint Gram contains the old source Gram, the carrier Gram and every phase/carrier/row mixed entry. One encloses this entire joint matrix, imposes the exact rows and tests its signed finite inequality. Both organizations use the same complete operator approximation and the same original-domain return; they retain their actual coefficient normalization.

The central computational resource in this paper is the uniform estimate (4), together with a whole-operator positive rule and a certified ambient threshold. The reader package reproduces these estimates from (1)–(3), including the full interval cover. In applications, (5) specifies the complete finite matrix condition to be checked with matched data.

A Theta and complex integration bounds

For $u \geq 0$ and $v = \pi e^{2u}$, the theta series is

$$\psi(u) = e^{u/2-v} \sum_{n \geq 1} (4n^4 v^2 - 6n^2 v) e^{-(n^2-1)v}.$$

Every real atom is positive. Dropping its negative term and using $\sum_{n \geq 1} n^4 e^{-3(n^2-1)} < 2$ gives the complete envelope

$$s(u)^2 \leq 169 \exp(4|u| - 3e^{2|u|}). \quad (13)$$

For example $\cosh(u/2) \geq e^{u/2}/2$ and $16\pi^2 < 169$ give the coefficient. Substitution $x = e^{2u}$ in its full two-sided integral yields

$$\int_{|u| > R} s(u)^2 du \leq 169(e^{2R}/3 + 1/9)e^{-3e^{2R}}.$$

This proves the spatial estimates used in the main text, with the slightly enlarged coefficient 240 in (10).

Real evaluations retain eight scaled atoms. For $v \geq \pi > 3$, the complete scaled remainder from $n \geq 9$ is bounded by

$$8\pi^2 9^4 e^{-80\pi} < 80 9^4 e^{-240} < 2^{-315}.$$

To obtain the first bound, maximize $v^2 e^{-(n^2-1)v}$ on $v \geq \pi$, and sum the ratio-bounded tail of $n^4 e^{-\pi(n^2-1)}$. This is an absolute scaled bound, restored before the positive denominator is evaluated.

For the physical carrier Gram use the strip $|\Im z| \leq 1/200$. Factoring the first atom as $2v(2v - 3)e^{z/2-v}$ bounds its phase by $(13/2)|\Im z| + 2\pi e^{2\Re z}|\Im z|$. The remaining atoms have absolute relative sum below $1/10$: $\Re v > 2.99$, and

$$3 \sum_{n \geq 2} n^4 e^{-(n^2-1)\Re v} \leq 3 \sum_{n \geq 2} n^4 (19^{-3})^{n-1} < 1/10.$$

On $0 \leq \Re z \leq 7/5$ the first-atom phase is less than one; the real part of ψ stays positive. Outside this split the complete absolute envelope

$$|\psi(x + iy)| \leq 120 e^{9|x|/2 - \pi e^{2|x|} \cos(1/100)}$$

is decreasing and is below $\alpha/4$ for $\alpha \geq 10^{-14}$. Direct summation on the strip also gives $|\psi| < 100$. Thus

$$\Re\{\alpha \cosh(z/2) + \psi(z)/5\} > \alpha/2, \quad |t_\alpha(z)^2| < 4 \cdot 10^{19}.$$

Even continuation pays negative real parts. The full-cell scalar comparisons are reproduced by the code.

B Complete carrier Gram arithmetic

Use 100 cells of halfwidth $1/80$ on $[-5/2, 0]$, each with 256 Gaussian nodes, and reflect. The parameter-7/5 ellipse has imaginary height $3/700 < 1/200$. Carrier frequencies are below 6000 and their weight factors are below one. The common holomorphic integrand bound is

$$M = 4 \cdot 10^{19} e^{12000/200}.$$

The complete reflected Gaussian error per Gram entry is at most

$$8(5/2)M(7/5)^{-512}/(1 - 5/7) < 10^{-25}.$$

Multiplication by 352 bounds its full operator cost. The whole physical exterior is positive and its trace is bounded using (10) and the total literal weight.

Arb encloses each scalar sine, cosine, square-root and theta value before binary64 conversion. The complete sample matrix Y has $n = 256 \cdot 100 \cdot 352$ entries. The guarded scalar conversions and at most six elementary operations bound its per-entry error by 2^{-45} . Its computed entries have modulus below one. Hence $\|\Delta Y\|_F \leq \sqrt{n} 2^{-45}$, and the sample-perturbation Gram cost is at most

$$(\sqrt{n} 2^{-45})(2\sqrt{n} + \sqrt{n} 2^{-45}).$$

Under the standard round-to-nearest binary64 arithmetic model, the dot products, 100 block additions and symmetrization cost at most $\gamma_{360}n$, where $\gamma_k = ku/(1 - ku)$ and $u = 2^{-53}$. These bounds include composition of the summation errors [5]. The analytic, physical and finite costs sum to the radius stated in Section 5.

A numerical eigensystem only proposes a fixed dyadic congruence or Rayleigh vector. Arb verifies invertibility of the congruence and every strictly positive Gershgorin row of the transformed upper test. The lower test evaluates the exact dyadic quadratic form in $\widehat{K} - \varepsilon_K I$ and divides by its exact positive vector norm. Program gates raise exceptions on failure, including under Python optimization.

C Reader reproduction and computational assistance

The package contains the complete cover, density rule, complementary data and two threshold endpoints, their portable producers, and a reproduction entry point. The cover is generated from the full symbol, the density rule from its actual positive set, and all subsequent enclosures from those literal data. Each acceptance test states its analytic contract. A clean run in normal and optimized Python reproduces the mathematical fields and threshold gates; a prescribed insufficient cap is rejected. The build entry point compiles this TeX source with Tectonic.

AI assistants supported derivation, programming and verification. The author is responsible for the statements and exposition. The mathematical arguments and complete reproducing code specify the computational evidence.

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